2nd Sem. **Method of Undetermined Coefficients**

We consider the problem of solving an equ^n of the form $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = R(x) \longrightarrow (1)$ for those cases in which the complete primitive of the reduced equ^n (making R.H.S. $R(x)=0$)

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0 \longrightarrow (2)$$

is known.

The complete primitive of (1) is

$$y = C.F + y_p(x)$$

where C.F means complementary function and $y_p(x)$ is any particular solution of (1).

To find y_p, we proceed as follows:

I. When the R.H.S $R(x)$ is an exponential: $R(x) = e^{ax}$

(i) ~~Whean~~ When a is not a root of the auxiliary equ^n i.e, e^{ax} is not in the C.F, take $y_p = Ae^{ax}$

Example :- Solve by the method of undetermined coefficients the following differential equ^n : $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 8e^{2x}$

Solution:- The given equ^n is $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 8e^{2x}$.

or, $(D^2 + 2D + 2)y = 8e^{2x}$ if $D \equiv \frac{d}{dx}$ $\longrightarrow (1)$

The auxiliary equ^n (A.E) is $m^2 + 2m + 2 = 0$

which gives $m = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm i$

$\therefore$ C.F $= e^{-x}(c_1 \cos x + c_2 \sin x)$

Here $a = 2$ is not a root of the A.E

So we take a particular integral (P.I) y_p in the form $y_p = Ae^{2x}$

diff^n both side w.r.t x

$D(y_p) = 2Ae^{2x}$

Again diff^n both side w.r.t x

$D^2(y_p) = 4Ae^{2x}$

putting the values of y_p, $D(y_p)$ and $D^2(y_p)$ in (1)

we get $D^2 y_p + 2Dy_p + y_p = 8e^{2x}$.

$$4Ae^{2x} + 2.2Ae^{2x} + 2Ae^{2x} = 8e^{2x}$$

or, $e^{2x}(10A) = 8e^{2x}$

$\therefore 10A = 8 \quad [\because e^{2x} \neq 0]$

or, $A = \frac{8}{10} = \frac{4}{5}$.

$\therefore y_p = \frac{4}{5} e^{2x}$

Hence the complete primitive is

$$\boxed{y = e^{-x}(c_1 \cos x + c_2 \sin x) + \frac{4}{5} e^{2x}}$$

where c_1 and c_2 are two arbitury constants.

(ii) when a is a simple root of A.E i.e, e^{ax} occurs in the C.F, assume $y_p = Axe^{ax}$

Example

Solve by the method of undetermined coefficients:

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 20e^{-2x}$$

Solution:- The given equ^n is $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 20e^{-2x}$.

$$\text{or, } (D^2 - D - 6)y = 20e^{-2x} \longrightarrow (1)$$

The A.E of (1) is $m^2 - m - 6 = 0$

or, $m^2 - 3m + 2m - 6 = 0$.

or, $m(m-3) + 2(m-3) = 0$

or, $(m+2)(m-3) = 0$.

i.e either $m = -2$ or $m = 3$.

$\therefore$ C.F $= c_1e^{-2x} + c_2e^{3x}$

Since $m = -2$ is a simple root of A.E

So we assume $y_p = Axe^{-2x}$

Now, diff^n both side w.r.t x, we get

$$D(y_p) = A \cdot \frac{d}{dx}\left[xe^{-2x}\right]$$

$$= A\left[x(-2)e^{-2x} + e^{-2x} \cdot \frac{d}{dx}(x)\right]$$

$$= A\left[-2xe^{-2x} + e^{-2x}\right]$$

$$= A(1-2x)e^{-2x}$$

Again diff^n both side w.r.t x, we get

$$D^2(y_p) = A \frac{d}{dx}\left[(1-2x)e^{-2x}\right]$$

$$= A\left[(1-2x)\frac{d}{dx}e^{-2x} + e^{-2x}\frac{d}{dx}(1-2x)\right]$$

$$= A\left[(1-2x)(-2)e^{-2x} + e^{-2x}(-2)\right]$$

$$= A\left[(1-2x)(-2)e^{-2x}\right.$$

or, $D^2(y_p) = A\left[-2e^{-2x}+4xe^{-2x}-2e^{-2x}\right]$

$= A\left[4xe^{-2x}-4e^{-2x}\right] = 4A(x-1)e^{-2x}$

putting the values of y_p, $D(y_p)$ and $D^2(y_p)$ in

(1), we get $D^2y_p - D y_p - 6y_p = 20e^{-2x}$

$4A(x-1)e^{-2x} - A(1-2x)e^{-2x} - 6Axe^{-2x} = 20e^{-2x}$

or, $e^{-2x}\left[4Ax-4A-A+2Ax-6Ax\right] = 20e^{-2x}$

or, $\left[6Ax-6Ax-5A\right] = 20 \quad \left[\because e^{-2x} \neq 0\right]$

or, $-5A = 20$

or, $A = -4$

so, $y_p = -4xe^{-2x}$

$\therefore$ Complete solution is $y = C_1e^{-2x}+C_2e^{3x}-4xe^{-2x}$

Where C_1 and C_2 are two arbitary constants.

(iii) when a is a double root of the A.E i.e, e^{ax} and xe^{ax} occur in the C.F, we take $y_p = Ax^2e^{ax}$.

Example:-

Solve by the method of undetermined coefficients:

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 24e^{-3x}.$$

Solution:- The given eqn is $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y = 24e^{-3x}$

or, $(D^2+6D+9)y = 24e^{-3x}$, if $D \equiv \frac{d}{dx}$ $\longrightarrow$ (1)

The A.E of (1) is $m^2+6m+9=0$

or, $(m+3)^2=0$

i.e, $m=-3,-3$.

$\therefore$ ~~C.F = ope c_1+c_2x~~

$\therefore$ $C.F=(c_1+c_2x)e^{-3x}$

Here $m=-3$ is a double root of the A.E

So we take $y_p=Ax^2e^{-3x}$.

Now, diff^n both side w.r.t x

$$\therefore D(y_p)=A\left[\frac{d}{dx}(x^2e^{-3x})\right]$$

$$=A\left[2xe^{-3x}+(-3)x^2e^{-3x}\right]$$

$$=A(2x-3x^2)e^{-3x}$$

Again diff^n both side w.r.t x, we get

$$D^2(y_p)=A\left[\frac{d}{dx}(2x-3x^2)e^{-3x}\right]$$

$$=A\left[(2-6x)e^{-3x}+(2x-3x^2)(-3)e^{-3x}\right]$$

$$=A\left[2-12x+9x^2\right]e^{-3x}.$$

putting the values of y_p, $D(y_p)$ and $D^2(y_p)$ we get

$$D^2y_p+6Dy_p+9y_p=24e^{-3x}$$

$$(9x^2-12x+2)Ae^{-3x}+6A(2x-3x^2)e^{-3x}+9Ax^2e^{-3x}=24e^{-3x}$$

or, $e^{-3x}\left[\cancel{9x^2}-\cancel{12x}+2+\cancel{12x}-\cancel{18x^2}+\cancel{9x^2}\right]A=24e^{-3x}$

or, $2A=24$ $\left[\because e^{-3x}\neq 0\right]$

or, $A=12$

$\therefore y_p = 12x^2 e^{-3x}$

Hence the complete solution is

$$\boxed{y = (c_1 + c_2 x)e^{-3x} + 12x^2 e^{-3x}}$$

Where c_1 and c_2 are two arbitary constants.

II. When the R.H.S $R(x)$ is a polynomial or a constant i.e, $R(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$

(A) Take $y_p = A_0 + A_1 x + A_2 x^2 + \cdots + A_n x^n$

In particular, if $R(x) = a_0$, a constant only, assume $y_p = A_0$

Example:-

(1) Solve by the method of undetermined coefficien

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = x^2 - 2x + 2$$

Solⁿ:- The given eqⁿ is $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = x^2 - 2x + 2$

or, $(D^2 - 4D + 1)y = x^2 - 2x + 2$

where $D \equiv \frac{d}{dx}$ $\longrightarrow$ (1)

The A.E of (1) is $m^2 - 4m + 1 = 0$

which gives $m = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$

$\therefore$ C.F $= c_1 e^{(2+\sqrt{3})x} + c_2 e^{(2-\sqrt{3})x}$

Here the R.H.S of ① is a polynomial of degree 2

So we can take the particular solⁿ y_p as the

form $y_p = A + Bx + Cx^2$.

diffⁿ both side w.r.t x

$D(y_p) = B + 2Cx$

Again diffⁿ both side w.r.t x

$D^2(y_p) = 2C$.

putting the values of $D(y_p)$ and $D^2(y_p)$ in ①

we get $\quad D^2y_p - 4Dy_p + y_p = x^2 - 2x + 2$

$2C - 4(B + 2Cx) + A + Bx + Cx^2 = x^2 - 2x + 2$

or, $Cx^2 + (B - 8C)x + A - 4B + 2C = x^2 - 2x + 2$

Equating coefficients of like powers of x:

$C = 1$,

$B - 8C = -2$
or, $B - 8 = -2$
or, $B = 6$

$A - 4B + 2C = 2$
or, $A - 24 + 2 = 2$
or, $A = 24$.

So, $y_p = 24 + 6x + x^2$

Hence the general solution is

$$y = C_1 e^{(2+\sqrt{3})x} + C_2 e^{(2-\sqrt{3})x} + x^2 + 6x + 24$$

Where A and B are two arbitary constants.

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(B) When $Q(x) = 0$, take $y_p = x(A_0 + A_1x + A_2x^2 + \cdots + A_nx^n)$.

Example:-
Solve by the method of undetermined coefficients

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} = x^2.$$

Solution:- The given eqn is $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} = x^2$

$$\text{or, } (D^2 - 4D)y = x^2 \text{ where } D \equiv \frac{d}{dx}. \quad \text{--- (1)}$$

The A.E of (1) is $m^2 - 4m = 0$

i.e, $m(m-4) = 0$

$\therefore m = 0, 4$

$\therefore$ C.F $= C_1 + C_2 e^{4x}$

Here the coefficient of y is 0. So we take the particular integral as $y_p = x(A_0 + A_1x + A_2x^2)$ --- (2)

diff^n ② w.r.t x, we get

$$D(y_p) = x(\cancel{A_0}\, A_1 + 2A_2 x) + A_0 + A_1 x + A_2 x^2$$

$$= A_0 + 2A_1 x + 3A_2 x^2$$

Again diff^n both side w.r.t x, we get

$$D^2(y_p) = 2A_1 + 6A_2 x$$

putting the values of $D(y_p)$ and $D^2(y_p)$ in ①, we get

$$D^2 y_p - 4Dy_p = x^2$$

$$2A_1 + 6A_2 x - 4(A_0 + 2A_1 x + 3A_2 x^2) = x^2$$

or, $(2A_1 - 4A_0) + (6A_2 - 8A_1)x - 12A_2 x^2 = x^2$

Equating the coefficients, we get

$2A_1 - 4A_0 = 0$	$6A_2 - 8A_1 = 0$	$-12A_2 = 1$
or, $A_0 = \frac{1}{2}A_1$	or, $A_1 = \frac{3}{4}A_2$	or, $A_2 = -\frac{1}{12}$
$= +\frac{1}{2}\left(-\frac{1}{16}\right)$	$= \frac{3}{4}\times\left(-\frac{1}{12}\right)$	
$= -\frac{1}{32}$	$= -\frac{1}{16}$	

$$\therefore y_p = x\left(-\frac{1}{32} - \frac{1}{16}x - \frac{1}{12}x^2\right)$$

The general sol^n is $y = c_1 + c_2 e^{4x} - x\left(\frac{1}{32} + \frac{1}{16}x + \frac{1}{12}x^2\right)$, c_1 and c_2 are arbitary constants

III. The given equ^n is $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = R(x)$

The complete primitive will be taken as

$$y = C.F + y_p$$

where y_p is any particular solution of the given equ^n.

When the R.H.S $R(x)$ Contains ~~the~~ Sines and Cosines,

i.e, $R(x) = \sin ax$, or, $\cos ax$

(A) When $\sin ax$ or $\cos ax$ is not in the C.F, take

$$y_p = A\sin ax + B\cos ax$$

For example:

Solve by the method of undetermined coefficients

$$\frac{d^2y}{dx^2} + 4y = 3\sin x$$

Sol^n:- The given equ^n is $\frac{d^2y}{dx^2} + 4y = 3\sin x$

or, $(D^2+4)y = 3\sin x \rightarrow (1)$

The A.E of (1) is $m^2+4 = 0$

which gives $m = \pm 2i$

$\therefore C.F = A\cos 2x + B\sin 2x$

Here $\sin x$ is not in C.F, we assume

$$y_p = A\sin x + B\cos x \longrightarrow (2)$$

Diff^n ② both side w.r.t 'x', we get

$$D(y_p) = A\cos x - B\sin x$$

Again diff^n both side w.r.t 'x', we get

$$D^2(y_p) = -A\sin x - B\cos x$$

putting the values of $D(y_p)$ and $D^2(y_p)$ in ①

we get $D^2y_p + 4y_p = 3\sin x$

$$-A\sin x - B\cos x + 4A\sin x + 4B\cos x = 3\sin x$$

or, $3A\sin x + 3B\cos x = 3\sin x$

Equating the coeff of $\sin x$ and $\cos x$, we get

$3A = 3$ and $3B = 0$

i.e. $A = 1$ i.e. $B = 0$.

$\therefore y_p = \sin x$

Hence the general sol^n is $y = A\cos 2x + B\sin 2x + \sin x$

where A and B are two arbitary constants.

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(B) When $\sin ax$ or $\cos ax$ is in the C.F, take

$$y_p = x(A\sin ax + B\cos ax).$$

For example.

Solve by the method of undetermined coefficients

$$\frac{d^2y}{dx^2} + 4y = \sin 2x$$

Solⁿ:- The given eqⁿ is $\frac{d^2y}{dx^2} + 4y = \sin 2x$

i.e, $(D^2+4)y = \sin 2x$ ⟶ ①

The A.E of ① is $m^2+4=0$.

which gives $m = \pm 2i$

$\therefore$ C.F $= A\cos 2x + B\sin 2x$

Since the R.H.S $\sin 2x$ appears in the Cf,

we take $y_p = x(A\cos 2x + B\sin 2x)$.

diffⁿ both side w.r.t 'x', we get

$$D(y_p) = A\cos 2x + B\sin 2x + x(-2A\sin 2x + 2B\cos 2x)$$

Again diffⁿ both side w.r.t. 'x', we get

$$D^2(y_p) = -2A\sin 2x + 2B\cos 2x + (-2A\sin 2x + 2B\cos 2x) + x(-4A\cos 2x - 4B\sin 2x)$$

or, $D^2(y_p) = -4A\sin 2x + 4B\cos 2x - 4x(A\cos 2x + B\sin 2x)$

putting the values of $D(y_p)$ and $D^2(y_p)$ in ①

$$D^2 y_p + 4y_p = \sin 2x$$

or, $-4A\sin 2x + 4B\cos 2x - 4x(A\cos 2x + B\sin 2x) + 4x(A\cos 2x + B\sin 2x)$

$$= \sin 2x$$

or, $-4A\sin 2x + 4B\cos 2x = \sin 2x + 0\cdot\cos 2x$.

Equating the coeff. of $\sin 2x$ and $\cos 2x$, we get

$-4A = 1$ and $4B = 0$

i.e. $A = -\frac{1}{4}$ i.e. $B = 0$

$\therefore y_p = -\frac{1}{4}x\cos 2x$.

Hence the general solⁿ is

$$y = A\cos 2x + B\sin 2x - \frac{1}{4}x\cos 2x.$$

Where A and B are two arbitary constants.

IV. When the R.H.S $R(x)$ contains products of an exponential, a polynomial, sines and cosines, that is

$R(x) = e^{ax}\sin bx$ or $e^{ax}(a_0 + a_1 x + \dots + a_n x^n)$ or, $\sin bx\,(a_0 + a_1 x + \dots + a_n x^n)$

etc. modify y_p accordingly with the help of I, II and III.

For example:

Solve by the method of undetermined coefficients:

$$\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 6y = (x-2)e^x$$

Solⁿ:- The given equⁿ is $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 6y = (x-2)e^x$

i.e, $(D^2 - 7D + 6)y = (x-2)e^x \longrightarrow (1)$

The A.E of (1) is $m^2 - 7m + 6 = 0$

or, $m^2 - 6m - m + 6 = 0$

or, $(m-1)(m-6) = 0$

$\therefore$ either $m = 1$ or $m = 6$

$\therefore C.F = c_1 e^x + c_2 e^{6x}$

Here we take $y_p = (Ax + Bx^2)e^x$

diffⁿ both side w.r.t 'x', we get

$D(y_p) = (Ax + Bx^2)e^x + (A + 2Bx)e^x$

$D(y_p) = (A + Ax + Bx^2 + 2Bx)e^x$

Again diffⁿ both side w.r.t 'x', we get

$D^2(y_p) = (A + 2Bx + 2B)e^x + (A + Ax + Bx^2 + 2Bx)e^x$

$\cancel{= (2B+2A) + (A+4B)x +}$

$\cancel{= 2A + A}$

$= (2A + 2B)e^x + (A + 4B)xe^x + Bx^2e^x$

putting these values in (1), we get

$(2A + 2B)e^x + (A + 4B)xe^x + Bx^2e^x - 7(A + Ax + Bx^2 + 2Bx)e^x$
$+ 6(Ax + Bx^2)e^x = (x-2)e^x$

or, $(2B - 5A)e^x + 4Bxe^x - 14Bxe^x = (x-2)e^x$

or, $(2B - 5A)e^x - 10Bxe^x = (x-2)e^x$

Equating the coefficients, we get

$2B - 5A = -2$ and $-10B = 1$

$\Rightarrow B = -\frac{1}{10}$

or, $5A = 2B + 2$

$= 2 - \frac{1}{5} = \frac{9}{5}$

or, $A = \frac{9}{25}$

$\therefore y_p = \left(\frac{9}{25}x + -\frac{1}{10}x^2\right)e^x$

Hence the general solution is $y = C_1e^x + C_2e^{6x} + \left(\frac{9}{25}x - \frac{1}{10}x^2\right)e^x$

Where C_1 and C_2 are two arbitrary constants.

Solve by the method of undetermined coefficients:

$(D^2 + D - 6)y = 10e^{2x} - 18e^{3x} - 6x - 11$

Solution :- The given equ^n is

$(D^2 + D - 6)y = 10e^{2x} - 18e^{3x} - 6x - 11 \longrightarrow (1)$

The A.E is $m^2 + m - 6 = 0$

$m^2 + 3m - 2m - 6 = 0$

or, $(m+3)(m-2) = 0$

$\therefore$ either $m = -3$ or $m = 2$

C.F $= C_1e^{-3x} + C_2e^{2x}$

Since $m = 2$ is a simple root of (1) so we take

$\therefore y_p = Axe^{2x} + Be^{3x} + Cx + D$

diff^n both side w.r.t 'x', we get

$D(y_p) = 2Axe^{2x} + Ae^{2x} + 3Be^{3x} + C$

Again diff^n both side w.r.t 'x', we get

$D^2(y_p) = 4Axe^{2x} + 2Ae^{2x} + 2Ae^{2x} + 9Be^{3x}$

$= 4Axe^{2x} + 4Ae^{2x} + 9Be^{3x}$

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putting the values of $D(y_p)$ and $D^2(y_p)$ in (1), we get

$$4Axe^{2x} + 4Ae^{2x} + 9Be^{3x} + 2Axe^{2x} + Ae^{2x} + 3Be^{3x} + C - 6(Axe^{2x} + Be^{3x} + Cx + D) = 10e^{2x} - 18e^{3x} - 6x - 11$$

or, $5Ae^{2x} + 12Be^{3x} - 6Be^{3x} - 6Cx - 6D + C = 10e^{2x} - 18e^{3x} - 6x - 11$

or, $5Ae^{2x} + 6Be^{3x} - 6Cx - 6D + C = 10e^{2x} - 18e^{3x} - 6x - 11$

$\cancel{6A = 10}$,

or, $\cancel{5Ae^{2x} + 6Be^{3x} - 6Cx - 6D + C = 10e^{2x} - 18e^{3x} - 6x - 11}$

$5A = 10$	$6B = -18$	$6C = 6$	$C - 6D = -11$
$A = 2$	$B = -3$	$C = 1$	or, $-6D = -12$
			or, $D = -2$

$$\therefore y_p = 2xe^{2x} - 3e^{3x} + x - 2$$

Hence the general sol^n is

$$y = C_1 e^{-3x} + C_2 e^{2x} + 2xe^{2x} - 3e^{3x} + x - 2$$

Where C_1 and C_2 are two arbitrary constants.

Solve by the method of undetermined coefficients:

$$(D^2 - 4D + 4)y = x^3 e^{2x} + xe^{2x}.$$

Sol^n:- The given equ^n is $(D^2 - 4D + 4)y = x^3 e^{2x} + xe^{2x}$. ... (1)

The A.E of (1) is $m^2 - 4m + 4 = 0$ which gives $m = 2, 2$

$$C.F = (C_1 + C_2 x)e^{2x}.$$

Here $m = 2$ is a double root of the A.E of (1)

So we take $y_p = x^2 e^{2x}(Ax^3 + Bx^2 + Cx + D)$

$$y_p = x^2 e^{2x}(Ax^3 + Bx^2 + Cx + D')$$

diff^n both side w.r.t x, we get

$$D(y_p) = x^2 e^{2x}(3Ax^2 + 2Bx + C) + 2xe^{2x}(Ax^3 + Bx^2 + Cx + D')$$
$$+ 2x^2 e^{2x}(Ax^3 + Bx^2 + Cx + D')$$

$$= e^{2x}\left[3Ax^4 + 2Bx^3 + Cx^2 + 2Ax^4 + 2Bx^3 + 2Cx^2 + 2D'x + 2Ax^5 + 2Bx^4 + 2Cx^3 + 2D'x^2\right]$$

$$= e^{2x}\left[(5A + 2B)x^4 + 2Ax^5 + (4B + 2C)x^3 + (3C + 2D')x^2 + 2D'x\right]$$

Again diff^n both side w.r.t 'x', we get

$$D^2(y_p) = e^{2x}\left[4(5A + 2B)x^3 + 10Ax^4 + 3(4B + 2C)x^2 + 2(3C + 2D')x + 2D'\right]$$
$$+ 2e^{2x}\left[(5A + 2B)x^4 + 2Ax^5 + (4B + 2C)x^3 + (3C + 2D')x^2 + 2D'x\right]$$

$$= e^{2x}\left[4Ax^5 + (20A + 4B)x^4 + (20A + 16B + 4C)x^3 + (12B + 12C + 4D')x + 6(C + D')x + 2D'\right]$$

putting the values of $D^2(y_p)$ and $D(y_p)$ in (1), we get

$$D^2y_p - 4Dy_p + 4y_p = x^3 e^{2x} + xe^{2x}$$

$$e^{2x}\left[4Ax^5 + (20A + 4B)x^4 + (20A + 16B + 4C)x^3 + (12B + 12C + 4D')x^2 + 6(C + D')x + 2D' - 4(5A + 2B)x^4 - 8Ax^5 - 4(4B + 2C)x^3 - 4(3C + 2D')x^2 - 8D'x + 4Ax^5 + 4Bx^4 + 4Cx^3 + 4D'x^2\right]$$
$$= x^3 e^{2x} + xe^{2x}.$$

or, $e^{2x}\left[20Ax^3 + 12Bx^2 + 6Cx - 2D'x + 2D'\right] = x^3 e^{2x} + xe^{2x}$

Equating the coefficients, we get

$20A = 1$ i.e, $A = \frac{1}{20}$	$12B = 0$ i.e, $B = 0$	$6C = 1$ or, $C = 1/6$	$2D' = 0$ i.e, $D' = 0$

$$\therefore y_p = x^2 e^{2x}\left[\frac{1}{20}x^3 + \frac{1}{6}x\right]$$

Hence the general solution is

$$y = (c_1 + c_2 x)e^{2x} + \left(\frac{1}{20}x^3 + \frac{1}{6}x\right)x^2 e^{2x}$$

where c_1 and c_2 are two arbitary constants.

Solve by the method of undetermined coefficients

$$(D^2 - 3D)y = x + e^x \sin x, \quad D \equiv \frac{d}{dx}$$

Solⁿ:-

The given equⁿ is $(D^2 - 3D)y = x + e^x \sin x \longrightarrow (1)$

The A.E of (1) is $m^2 - 3m = 0$

ie, $m(m-3) = 0$

$\therefore$ either $m = 0$ or $m = 3$

$$\therefore C.F = c_1 e^{0x} + c_2 e^{3x} = c_1 + c_2 e^{3x}.$$

We take $y_p = x(A + Bx) + Ce^x \sin x + D' e^x \cos x$

diffⁿ both side w.r.t 'x', we get

$$D(y_p) = A + 2Bx + C(e^x \sin x + e^x \cos x) + D'(e^x \cos x - e^x \sin x)$$

Again diffⁿ both side w.r.t 'x', we get

$$D^2(y_p) = 2B + C(e^x \sin x + e^x \cos x + e^x \cos x - e^x \sin x) + D'(e^x \cos x - e^x \sin x - e^x \sin x - e^x \cos x)$$

or, $D^2(y_p) = 2B + 2Ce^x \cos x - 2D' e^x \sin x$

putting the value $D(y_P)$ and $D^2(y_P)$ in ①, we get $D^2 y_p - 3D y_p = x + e^x \sin x$

$$2B + 2Ce^x \cos x - 2D'e^x \sin x - 3\left(A + 2Bx + C(e^x \sin x + e^x \cos x) + D'(e^x \cos x - e^x \sin x)\right)$$
$$= x + e^x \sin x.$$

or, $(2B - 3A) - 6Bx + (2C - 3C - 3D')e^x \cos x - \cancel{(2D + 3C - 3D)}$
$$(2D' + 3C - 3D')e^x \sin x = x + e^x \sin x$$

or, $(2B - 3A) - 6Bx - (C + 3D')e^x \cos x + (D' - 3C)e^x \sin x = x + e^x \sin x$

Comparing the coefficients, we get

$2B - 3A = 0$

or, $3A = 2B$

or, $A = \frac{1}{3}B$

$= \frac{2}{3} \times \left(-\frac{1}{6}\right)$

or, $A = -\frac{1}{9}$

$6B = -1$

or, $B = -1/6$

$C + 3D' = 0$ ... ⓐ

$D' - 3C = 1$ ⓑ

ⓐ $-$ 3ⓑ $\Rightarrow$

$C + 3D' - 3D' + 9C = 0 - 3$

or, $10C = -3$

$C = -3/10$

$D' = -\frac{C}{3} = -\frac{1}{3} \times \left(-\frac{3}{10}\right)$

$= \frac{1}{10}$.

$$\therefore y_p = x\left(-\frac{1}{9} - \frac{1}{6}x\right) - \frac{3}{10}e^x \sin x + \frac{1}{10}e^x \cos x$$

Hence the general solⁿ is

$$y = C_1 + C_2 e^{3x} - x\left(\frac{1}{9} + \frac{1}{6}x\right) - \frac{3}{10}e^x \sin x + \frac{1}{10}e^x \cos x$$
$$= C_1 + C_2 e^{3x} - \frac{1}{3}\left(\frac{x}{3} + \frac{x^2}{2}\right) + \frac{e^x}{10}(\cos x - 3 \sin x)$$

where C_1 and C_2 are two arbitrary constants.

Dbgie