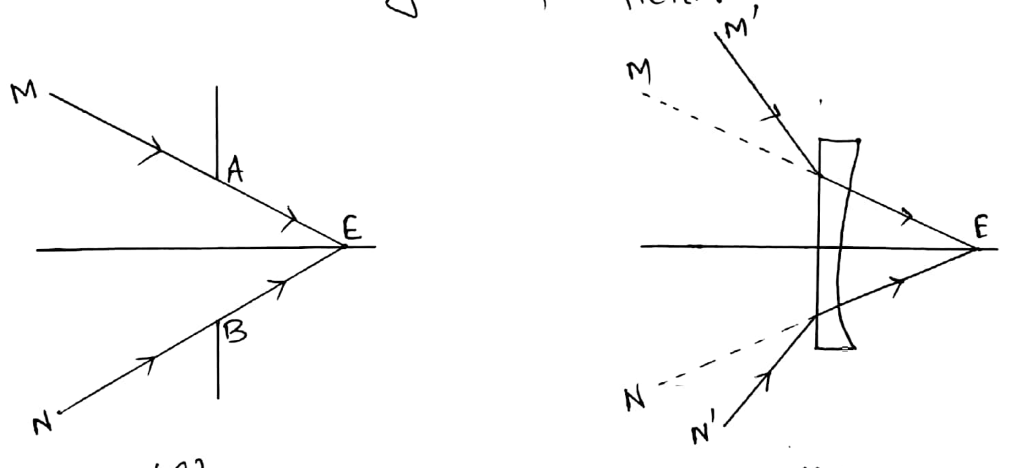
EYE - PIECE

Defination

It is a device used to magnify an image formed by a lens or lenses preceding it in any optical instrument. for example, the objective of a microscope or telescope forms a real image of an object under examination. The function of the eye-piece is to enlarge this image so that the final image is free from various aberrations as seen by the eye.

field of view

field of view of an optical instrument may be defined as the extend or range of its visual area. for an eye-piece it is the widest dimension of an object which is visible through the eye-piece. Generally, it is expressed as the width in feet at 1000 yards or in degree of field.



(a) (b)

The field of view is shown in fig: (a) and with a lens (fig: b). When the field of view is expressed in feet, it is called the linear field of view. When it is shown in degree, it is known a angular field of view. We can convert one to other by using a formula, 1 angular degree of view = 52.5 feet.

As shown in figure light is passing through the aperture AB and reaching the eye E. The eye can view objects in the angle range ∠MEN. The widest possible angle ∠MEN which can see, is called field of view. If a concave lens is added in plane AB from a wider field M'EN' can reach the eye. We say the field of view is enlarged. On the otherhand convex lens narrow the field of view.

Need of Multiple lens eyepiece

The basic purpose of optical instrument is to produce a magnified image free from aberrations and enlarged field of view. A single lens fails to fulfill both the requirements as the image formed by single lens suffers from chromatic and spherical aberration. Also the field of view is small and with the increase in magnification of optical instruments it became even smaller as the image formed by marginal rays refracted through the peripheral portion of eye lens can't simultaneously enter the small aperture of the pupil of eye placed near the eye lens. Hence, only the part of image nearer to the axis will be seen and final image will cover a small field of view.

Field of view is also decreased with the increase in distance between the objective and eye lens. As the magnifying power depends on distance so field of view is also decreased as the instrument is set for higher magnifying power by adjusting the distance between objective lens and eyelens. Now to ~~overcome~~ overcome the shortcomings of a single lens the eye lens is replaced by multiple lens eyepiece in optical instruments. In general, in an eyepiece there are two lens made by suitable material and type

Seperated by suitable distance. The extra lens facing the objective of instruments is known a field lens. Field lens and eye lens are made and kept in such a way that their combination minimizes chromatic and spherical aberration. Also field lens gathers more of the rays from the objective towards the axis of the eyepiece and covers all the rays from the image to enter the eye lens. In this way it increases the field of view.

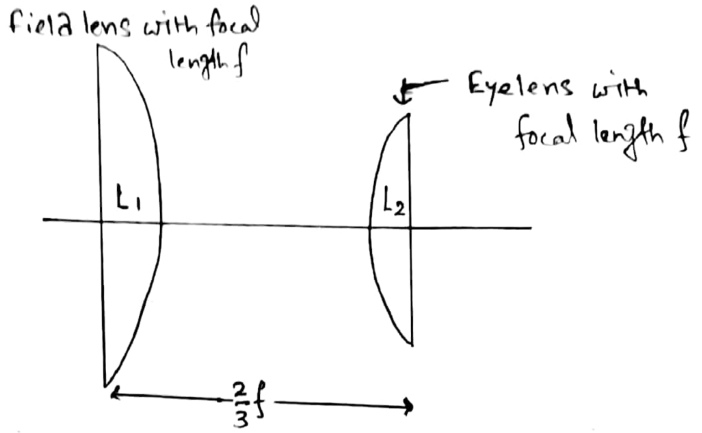
Positive and Negative Eyepiece

Eyepiece in which the first focal plane of an eyepiece lies in the object space outside the eyepiece is called positive eyepiece. In such eyepiece a real object can be placed on first focal plane to be in focus with final image i.e. Ramsden's eyepiece. Eyepiece in which the first focal plane of an eyepiece lies within eyepiece i.e. between eye lens and field lens where no real object can be placed is called negative Eyepiece. i.e: Huygen's Eyepiece

Ramsden's Eyepiece :→

Construction: Ramsden's Eyepiece consists of two planoconvex lenses made of same material with their curved (convex) side facing each other inwards (fig). In this type of eyepiece focal length of field lens and eye lens is same and distance between field lens and eye lens is $\frac{2}{3}f$ where f is focal length of eyelens.

The field lens is a little larger and is placed close to intermediate image to allow maximum



possible light to pass through it. The eye lens has a smaller diameter but caries out the actual magnification.

Condition of Achromatism :-

For achromatism the distance d between two lenses should be,

$$d = \frac{f_1 + f_2}{2}$$

In case of Ramsden's eyepiece $f_2 = f = f_1$

$$\therefore d = f$$

But the distance between field lens and eye lens in Ramsden's eyepiece is kept $\frac{2}{3}f$ which is slightly less than f, therefore, not completely free from chromatic aberration. If the distance between field lens and eye lens is kept f the field lens will be at the focal plane of eye lens. In this position any dust particle or scratch would be magnified and final image will not be clear.

Condition for minimum Spherical Aberration :-

The distance between the two lenses for minimum spherical aberration should be, $d = (f_1 - f_2)$

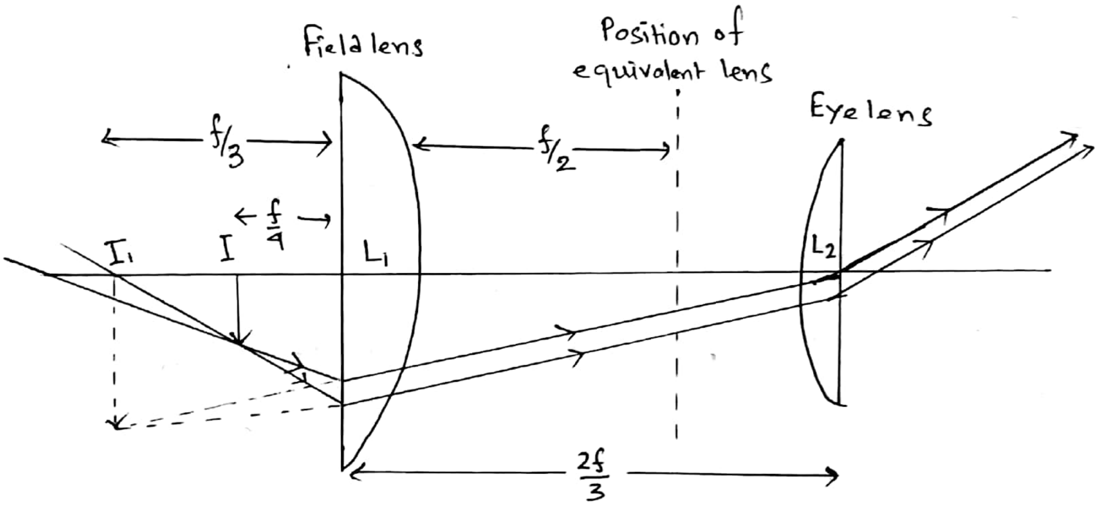
In case of Ramsden's eyepiece, $f_1 = f = f_2$

$$\therefore d = f - f = 0.$$

It means that it doesn't satisfy condition for minimum spherical aberration. However to reduce spherical aberration in Ramsden's eyepiece two plano-convex lenses with their convex lens facing each other are used.

Working

When eye piece is adjusted for normal vision, the final image formed by it is at infinity. For this the image formed, the field lens should lie in the first focal plane of the eye lens. The objective forms the real inverted image I of a distant object. This acts as an object for the field lens. This gives rise to a virtual image I_1. I_1 in turns serves as an object for the eye lens therefore it must be at distance equal to focal length f from eyes lens to make final image at infinity. Hence image I_1 lies at a distance $f/3$ to the left of the field lens.



For field lenses the image I_1 formed by the objective of the instrument (in which eye-piece is used) acts as an object for eye lens.

If v is the distance of I_1 from the field lens, than from lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Since, here $v = -f/3$ and $f = f$, we have,

$$\therefore \frac{1}{-f/3} - \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad \frac{1}{u} = -\frac{3}{f} - \frac{1}{f} = -\frac{4}{f}$$

$\therefore$ $\boxed{u = -f/4}$ $\Rightarrow$ That is, the image I formed by objective lies to the left of the field lens. The rays coming from I_1 after ~~coming~~ emerging from the field lens appear to come from I_1 at a distance $f/3$ left to field lens. These rays emerge from the eye lens as a parallel beam.

Equivalent Focal Length :→

Equivalent focal length of Ramsden's eyepiece is,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$= \frac{1}{f} + \frac{1}{f} - \frac{2/3 f}{f^2}$$

$$= \frac{2}{f} - \frac{2}{3f} = \frac{4}{3f}$$

$$\therefore \boxed{F = \frac{3}{4} f}$$

When eyepiece is adjusted for normal vision, the final image formed by it, should be at infinity. For this the image formed by the equivalent lens must be at infinity. For this I_1 should lie in the first focal plane of the equivalent lens.

Cardinal Points of Ramsden's Eyepiece :→

Position of focal points

The distance of first focal point F_1 from the field lens L_1 is given by,

$$\beta_1 = -F\left(1 - d/f_2\right)$$

$$= -\frac{3}{4}f\left(1 - \frac{2f/3}{f}\right) = -\frac{3}{4}f \times \frac{1}{3} = -\frac{f}{4}$$

Where negative sign indicates that first focal point F_1 lies at a distance of $(f/4)$ to the ~~right of eyelens L_2~~. left of field lens L_1. The distance of second focal point F_2 from the eye lens L_2 is given by.

$$\beta_2 = F\left(1 - d/f_2\right) = \left(\frac{3}{4}f\right)\left(1 - \frac{2f/3}{f}\right) = \frac{3}{4}f \times \frac{1}{3} = +\frac{f}{4}.$$

Where positive sign indicates the second focal point F_2 lies at a distance $(f/4)$ to the right of eye lens L_2.

Position of Principal Points :-

The distance of first principal point H_1 from the field lens L_1 is given by,

$$\alpha_1 = F \cdot \frac{d}{f_2} = \frac{\left(\frac{3}{4}f\right)\left(\frac{2}{3}f\right)}{f} = +\frac{f}{2}$$

Where positive sign indicates that the first principal point H_1 lies at a distance of $f/2$ to the right of the field lens L_1.

The distance of second principal point H_2 from the eye lens L_2 is given by,

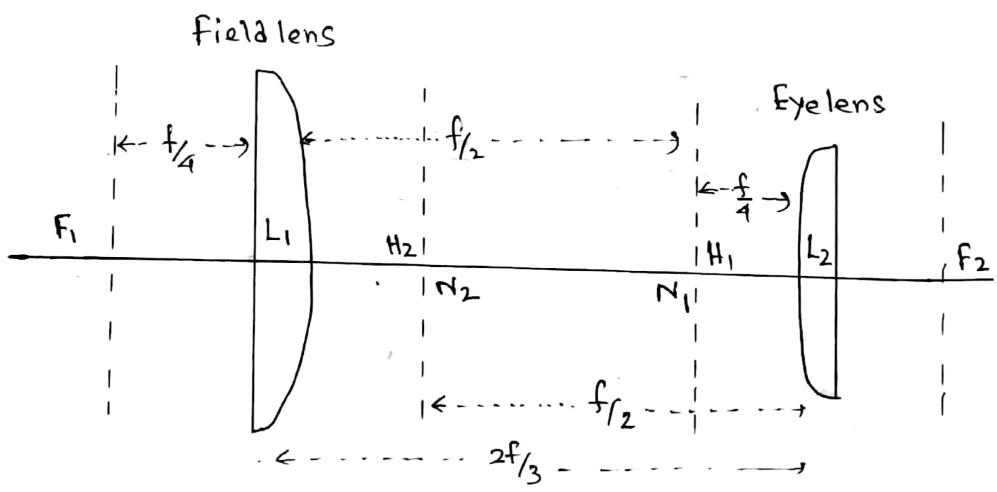
$$\alpha_2 = -F \cdot \frac{d}{f_1} = -\frac{\left(\frac{3}{4}f\right)\left(\frac{2}{3}f\right)}{f} = -\frac{f}{2}$$

Where negative sign indicates that the second principal point H_2 lies at a distance of $(f/2)$ to the left of the eye lens L_2.

Position of Nodal Points :-

As the medium on either side of the eyepiece is same (air), the nodal points N_1, N_2 coinside with the principal points H_1, H_2 respectively.

The position of cardinal points for Ramsden's eyepiece can be plotted as,



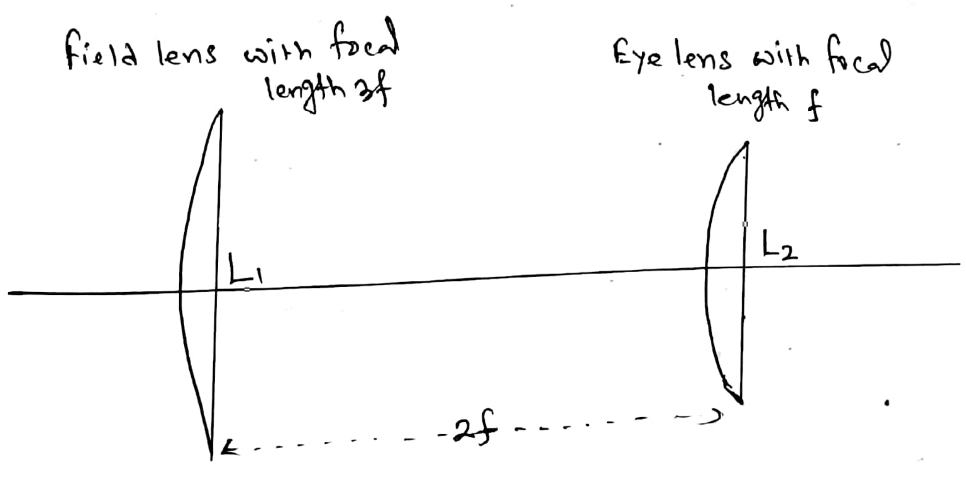
Position of Cross wires :-

The cross wires, if used, must be placed at the position where the image due to objective is formed so that they would be in focus with the final image and magnified by both the lens of eyepiece. The cross wires placed in first focal plane satisfies both the conditions. Therefore cross wire must be placed at a distance of $(f/4)$ in front of field lens. As it is outside both the lenses in object space and is real hence it is used to examine a real object or real image hence it is called a positive eyepiece. Therefore this eyepiece is used in optical instruments where accurate quantitative measurements of distances and angles are made.

Huygen's Eyepiece

Construction

Huygen's eyepiece consist of a combination of two coaxial plano convex lenses having focal length in the ratio 3:1 seperated by the distance between them is equal to the difference in their focal length. The focal length and the positions of the two lenses are such that the system is free from chromatic as well as spherical aberrations. The field and eye lenses are placed with their convex surface towards the incident ray



Condition of Achromatism :-

For achromatism the distance d between two lenses should be,

$$d = \frac{f_1 + f_2}{2}$$

In case of Huygen's eyepiece, $f_1 = 3f$, $f_2 = f$.

$$\therefore \quad d = \frac{3f + f}{2} = 2f.$$

This is the distance between field lens and eye lens. Hence. Huygen's eyepiece is free from chromatic aberration.

Condition for minimum Spherical Aberration :-

The distance between the two lenses for minimum spherical aberration should be,

$$d = f_1 - f_2$$

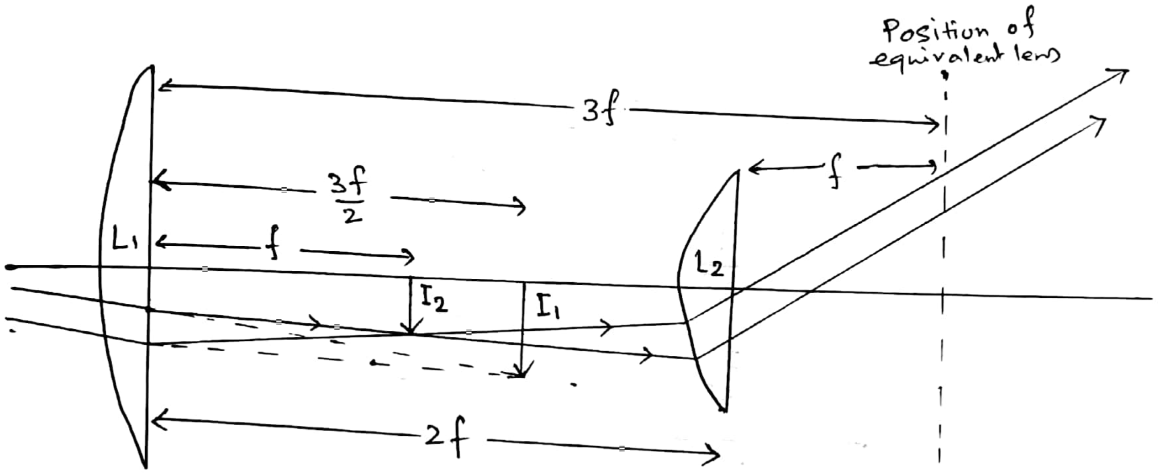
In case of Huygen's eyepiece, $f_2 = 3f$, $f_1 = f$

$$\therefore \quad d = 3f - f = 2f.$$

This is the distance between field and eye lens in Huygen's eyepiece hence huygen's eyepiece is free from spherical aberration.

Working

Like Ramsden's eyepiece when eye piece is adjusted for normal vision, the final image formed by it is at infinity. For this the image formed by the field lens should lies in the first focal plane of the eye lens i.e at a distance f to the left of eye lens or at a distance f to the right of field lens as the distance between eye and field lens is 2f. The inverted image I_1 of a distant object, formed by objective, acts as an object for the field lens. This gives rise to a virtual image I_2. I_2 in turn serves as an object for the eye lens therefore it must be at distance equal to focal length f from eye lens to make final image at infinity.



$$\therefore \frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

Since $v = f$ and $f = 3f$ we have,

$$\frac{1}{f} - \frac{1}{u} = \frac{1}{3f}$$

$$\text{or,} \quad \frac{1}{u} = \frac{1}{f} - \frac{1}{3f} = \frac{2}{3f}.$$

$$\therefore \boxed{u = \frac{3f}{2}}$$

The positive sign indicates that image I_1 formed by field lens as well as image I_2 formed by objective lies on the same side i.e. the field lens focused the rays at I_2 which otherwise would be focused at I_1 by objective. The rays coming from I_2 emerge from the eye lens as a parallel beam

Equivalent focal length :-

Equivalent focal length of Huygen's eyepiece is given by.

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

Here, $f_1 = 3f$, $f_2 = f$ and $d = 2f$

$$\therefore \frac{1}{F} = \frac{1}{3f} + \frac{1}{f} - \frac{2f}{3f^2}$$

$$= \frac{4}{3f} - \frac{2}{3f}$$

$$= \frac{2}{3f}$$

$$\therefore \boxed{F = \frac{3f}{2}}$$

When eyepiece is adjusted for normal eye vision, the final image formed by it should be at infinity. For this, the image formed by the equivalent lens must be at infinity. For this, equivalent lens must be placed at a distance $(3f/2)$ to the right of f_1 or at a distance f to the right of eye lens.

Cardinal Points of Huygen's Eyepiece :-

Position of focal points :-

The distance of first focal point F_1 from the field lens L_1 is given by,

$$\beta_1 = +F\left(1 - \frac{d}{f_2}\right) = -\left(\frac{3f}{2}\right)\left(1 - \frac{2f}{f}\right) = \frac{3f}{2}$$

Positive sign indicates that first focal point F_1 lies at a distance of $\left(\frac{3f}{2}\right)$ to the right of field lens L_1.

The distance of second focal point F_2 from the eye lens L_2 is,

$$\beta_2 = F\left(1 - \frac{d}{f_1}\right) = \left(\frac{3f}{2}\right)\left(1 - \frac{2f}{3f}\right) = \frac{3}{2}f \times \frac{1}{3} = +\frac{f}{2}$$

Positive sign indicates that second focal point F_2 lies at a distance $(f/2)$ to the right of eye lens L_2.

Scanned by CamScanner

Position of Principal Points :-

The distance of first principal point H_1 from the field lens L_1 is given by,

$$\alpha_1 = f \cdot \frac{d}{f_2} = \left(\frac{3f}{2}\right) \frac{2f}{f} = +3f$$

Positive sign indicates that first principal point H_1 lies at a distance of $(3f)$ to the right of field lens L_1.

The distance of second principal point H_2 from the eye lens L_2 is given by,

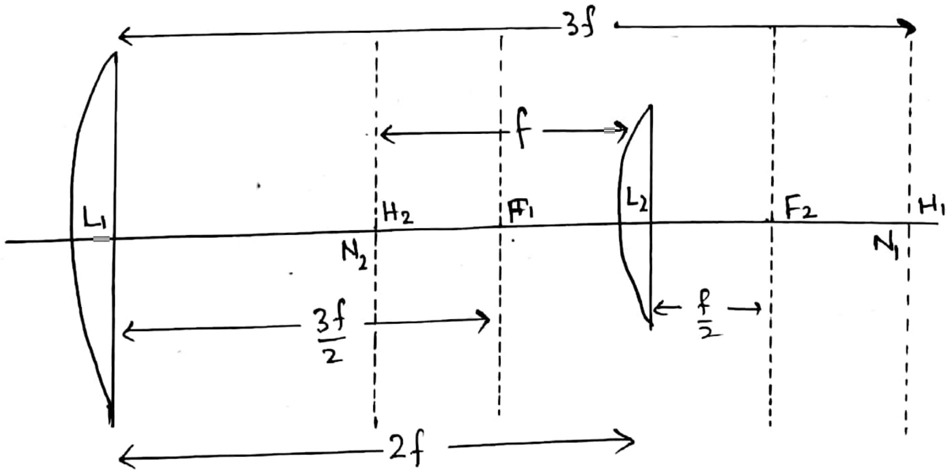
$$\alpha_2 = -F \cdot \frac{d}{f_1} = -\left(\frac{3f}{2}\right) \frac{2f}{3f} = -f$$

Negative sign indicates that second principal point H_2 lies at a distance of (f) to the left of eye lens L_2.

Position of Nodal points :-

As the medium on either side of the eyepiece is same (air), the nodal points N_1, N_2 coinside with the principal points H_1, H_2 respectively.

The position of cardinal points for Huygen's eyepiece can be plotted as :



Position of Cross wires

The cross wires if used must be placed at the position where the image due to objective is formed so that they would be in focus with the final image and magnified by both the lens of eyepiece. The cross wires placed in first focal plane satisfies both the conditions. However, in Huygen's eyepiece first focal plane lies between the lenses of eyepiece and is virtual. Therefore, cross wire must be placed at a distance of $\left(\frac{3f}{2}\right)$ in right side of field lens and $\left(\frac{f}{2}\right)$ left to eye lens. i.e. in between the lenses of eyepiece. So it is magnified by eye lens only while the image is magnified by both the lenses. It is called a negative eyepiece as the first focal plane lies within the eyepiece where no real objects can be placed. Therefore, this eyepiece is used to examine a virtual image like in microscope. Therefore, generally eyepiece is not used in this type of eyepiece.

Comparision of Ramsden's & Huygen's Eyepiece :-

(i) Field of view : The field of view in both these eyepiece is fairly large in the image space.

(ii) Achromatism & Spherical aberration :- Huygen's eyepiece is constructed so that it has minimum spherical aberration. Ramsden's eyepiece has also minimum spherical aberration as in this case the deviation of rays is shared by all the four surfaces of the two lenses comprising it.

The Huygen's eyepiece is free from chromatic aberration as, $d = \frac{f_1 + f_2}{2}$. However Ramsden's eyepiece does not satisfy this condition and show lateral colours. This effect can be reduced by using achromats in place of field and eye lenses

Huygen's eyepiece is achromatic for all colours whereas Ramsden's eyepiece is achromatic for only the selected colours.

(iii) In Huygen's eyepiece, if the crosswires are replaced by measuring scales, the measurements are not trustworthy due to unequal magnification. In Ramsden's eyepiece, this difficulty is not present and hence the measurements are trustworthy.

In Ramsden's eyepiece the field of view is much better than that in the case of Huygen's eyepiece.

(iv) The distance between the eye-lens and the eye ring is greater in Ramsden's eyepiece than in Huygen's eyepiece. Hence eye-relief is better in Ramsden's eye-piece.

Prefering Ramsden's eyepiece over Huygen's eyepiece:-

In Ramsden's eyepiece the crosswires are placed in front of the eye-piece and hence magnification of cross wire and image is the same, but in Huygen's eyepiece the cross wires are placed in between the field-lens and the eye-lens, hence magnification of image and cross wires is not the same. The measurements with Huygen's eyepiece are not trustworthy.

Problem 1. Two thin lenses of focal length 10 cm and 8 cm are kept in position of least spherical aberration. Find the distance of principal points for the system.

Problem: 2. A convergent doublet of seperated lenses has been corrected for spherical aberration. It has an equivalent focal length of 10 cm. Leses are seperated by 2 cm. What are the focal lengths of its component lenses?

Problem: 3. In Huygen's eyepiece focal length of field lens is 6 cm. Locate the positions of cardinal points in a diagram.

— o —