Engineering Physics 1
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Module-05
Lecture-03
Maxwellian Distribution Law of Velocity – Part 01

Today I will be giving lecture on the Maxwell distribution law of velocities. So, let me start it. Actually before coming to the topic let me tell you what are the random quantities and the probability. First of all I should familiarize with these quantities then I will start what is the derivation of Maxwell distribution law of velocities. Random quantities and probability let us consider a small region in the space occupied by an ideal gas.

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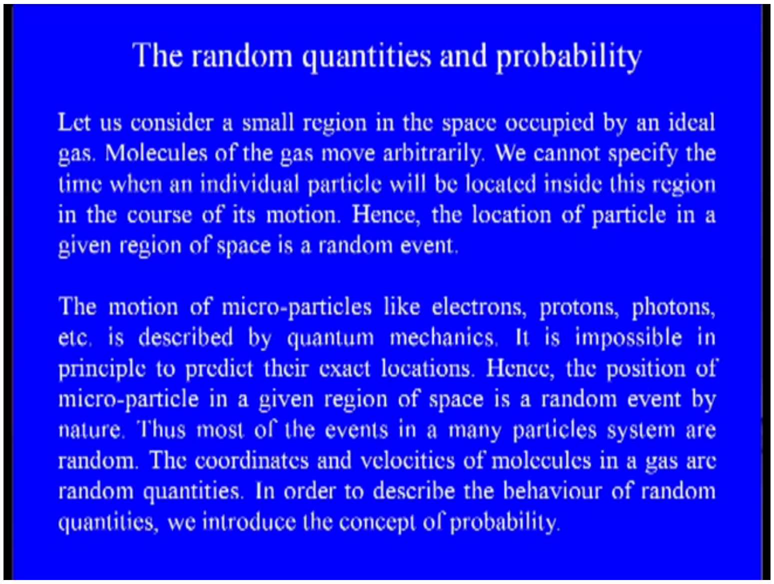

The random quantities and probability

Let us consider a small region in the space occupied by an ideal gas. Molecules of the gas move arbitrarily. We cannot specify the time when an individual particle will be located inside this region in the course of its motion. Hence, the location of particle in a given region of space is a random event.

The motion of micro-particles like electrons, protons, photons, etc. is described by quantum mechanics. It is impossible in principle to predict their exact locations. Hence, the position of micro-particle in a given region of space is a random event by nature. Thus most of the events in a many particles system are random. The coordinates and velocities of molecules in a gas are random quantities. In order to describe the behaviour of random quantities, we introduce the concept of probability.

Molecules of the gas move arbitrarily we cannot specify the time when an individual particle will be located inside this region in the course of its motion. Hence the location of a particle in a given region of space is a random event. The motion of micro particles like electrons, protons, photons etcetera is described by quantum mechanics not by classical mechanics. It is so it is impossible in principle to predict their locations as well as they are dynamic simultaneously.

Which is nothing but the uncertainty principle between position and momentum hence the position of a micro particle in a given region of space is a random event by nature. Suppose as we know according to the uncertainty principle any two conjugate variable cannot be measured simultaneously accurately. Suppose if I can localize a particle, suppose if I know the position of a particle then its momentum could be completely uncertain. So, that is the reason simultaneously if I know its dynamics. Suppose if I know its momentum then its position is completely uncertain.

So, that is the reason the position of micro particle in a given region of space is a random event by nature. Thus most of the events in a many particle system are random. The coordinates and velocities of the molecules in a gas are random quantities. In order to describe the behavior of random quantities we introduce the concept of probability. In fact in our lecture we will derive what is the probability of finding a molecule having velocities between V and v + dv in a gas.

So, that is the reason that is the quantity we are going to calculate now let me start what is Maxwell distribution law of velocity.

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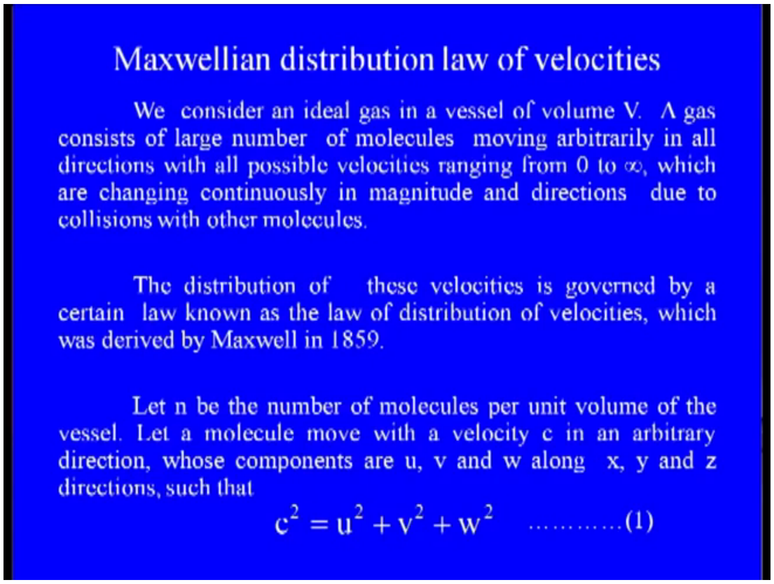

Maxwellian distribution law of velocities

We consider an ideal gas in a vessel of volume V. A gas consists of large number of molecules moving arbitrarily in all directions with all possible velocities ranging from 0 to ∞, which are changing continuously in magnitude and directions due to collisions with other molecules.

The distribution of these velocities is governed by a certain law known as the law of distribution of velocities, which was derived by Maxwell in 1859.

Let n be the number of molecules per unit volume of the vessel. Let a molecule move with a velocity c in an arbitrary direction, whose components are u, v and w along x, y and z directions, such that

$$c^2 = u^2 + v^2 + w^2 \qquad \ldots\ldots\ldots\ldots(1)$$

We consider an ideal gas in a vessel of volume V a gas consists of large number of molecules moving arbitrarily in all directions with all possible velocities ranging from 0 to infinity which are changing continuously in magnitude as well as directions due to collision with other molecules. This is the first assumptions of the derivation of Maxwell distribution law of velocities.

That means the velocities of the molecules in a gas can; will be from 0 to infinity they can take any velocity. The Infinity means maximum limit of the velocity of light is obviously velocity of light in free space. But as we have already told in the lecture series of kinetic theory of gases that that will not introduce any error. So, that is the reason for the sake of the calculations we are saying the velocity of the molecules in a gas can take values from 0 to infinity.

The distribution of these velocities is governed by a certain law known as the law of distribution of velocities. It will be known as the Maxwell distribution law of velocities obviously it was derived by Maxwell in 1859. Now let me start it by assuming n be the number of molecules per unit volume of the vessel and let a molecule move with a velocity c in an arbitrary directions whose components of along the x, y, z direction are u, v, w respectively.

And obviously so the length of this velocity vector c square = u square + v square + w square like the radius vector in the coordinate space is r square = x square + y square + z square obviously the radius is measured from the origin which is defined as the 0, 0 to x, y, z. Similarly here the in the velocity space the origin coincides with the 0 0 and the a point in the xyz plane is uvw, so, c square = u square + v square + w square.

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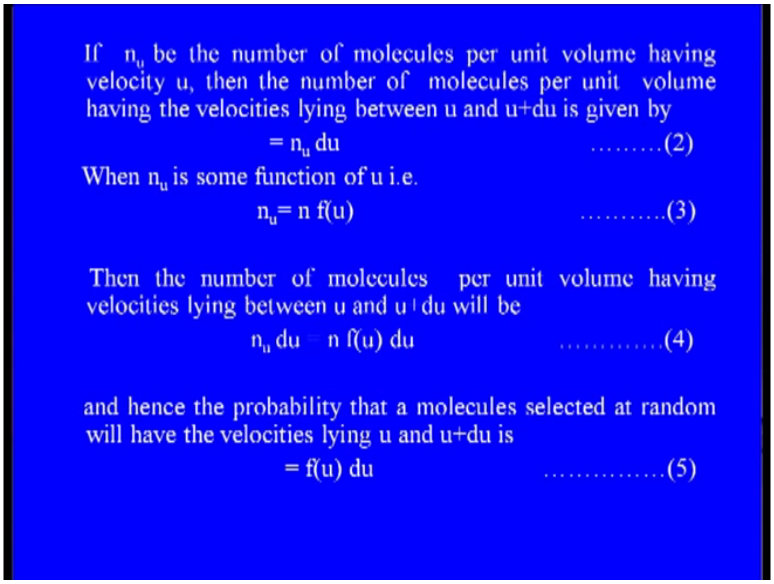

If n_u be the number of molecules per unit volume having velocity u, then the number of molecules per unit volume having the velocities lying between u and u+du is given by

$= n_u\,du$ ………(2)

When n_u is some function of u i.e.

$n_u = n\,f(u)$ ……….(3)

Then the number of molecules per unit volume having velocities lying between u and u+du will be

$n_u\,du = n\,f(u)\,du$ …………(4)

and hence the probability that a molecules selected at random will have the velocities lying u and u+du is

$= f(u)\,du$ …………..(5)

If nu the number of molecules per unit volume having velocity u then the number of molecules per unit volume having the velocities lying between u and u + du is given as a nu du. So, this is the first thing if nu be the number of molecules having velocities u and u + du. Then the number of molecules per unit volume having the velocities lying between u and u + du is a nu du ok. When nu is some function of u then nu du is f of u du ok.

Then the number of molecules per unit volume having velocities lying between u and u + du will be nu du = n into fu du and hence the probability that a molecule selected at random will have the velocities lying between u and u + du is fu du. Basically you have to divide the equation 4 by a so you will get the probability which is nothing but fu du.

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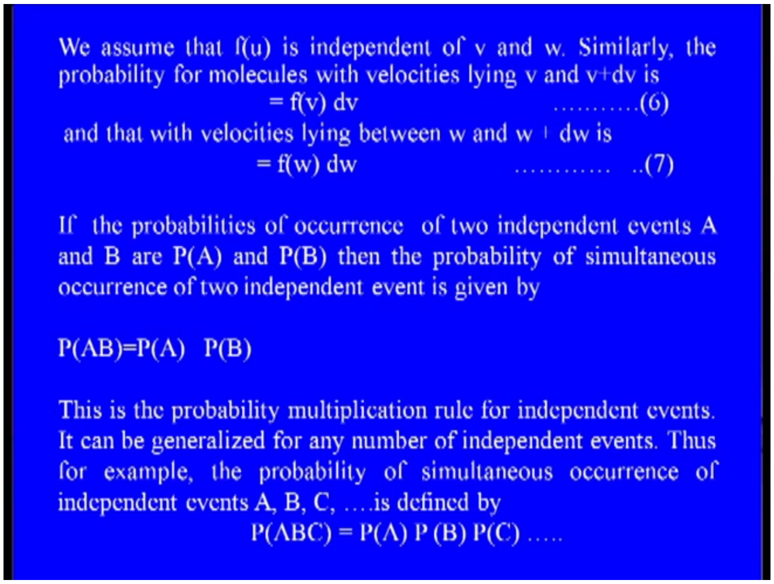

We assume that f(u) is independent of v and w. Similarly, the probability for molecules with velocities lying v and v+dv is

$$= f(v)\,dv \qquad \text{..........(6)}$$

and that with velocities lying between w and w + dw is

$$= f(w)\,dw \qquad \text{............ ..(7)}$$

If the probabilities of occurrence of two independent events A and B are P(A) and P(B) then the probability of simultaneous occurrence of two independent event is given by

$$P(AB)=P(A)\ \ P(B)$$

This is the probability multiplication rule for independent events. It can be generalized for any number of independent events. Thus for example, the probability of simultaneous occurrence of independent events A, B, C, ….is defined by

$$P(ABC) = P(A)\,P(B)\,P(C)\,\ldots..$$

We assume that a fu is independent of v and w similarly the probability for molecules with velocities lying between v and v + dv is a vdv and the probability of finding the molecules having velocities u w and w + dw is f of w dw. Here we have assumed that these separate these events of uvw these are independent to each other. Then we will use a theorem in the probability theory that I am going to tell it now.

If the probabilities of occurrence of two independent events A and B are PA and PB, then the probability of simultaneous occurrence of two independent event is given by P of AB = it is a product of two probabilities PA times PB. This is the probability multiplication rule for independent events. It can be generalized for any number of independent events.

Thus for example the probability of simultaneous occurrence of independent events let us say ABC is defined as P of ABC which is equal to the product of multiple product of individual events so equal to PA into PB into PC etcetera. So, now we will use this probability theorem in our case.

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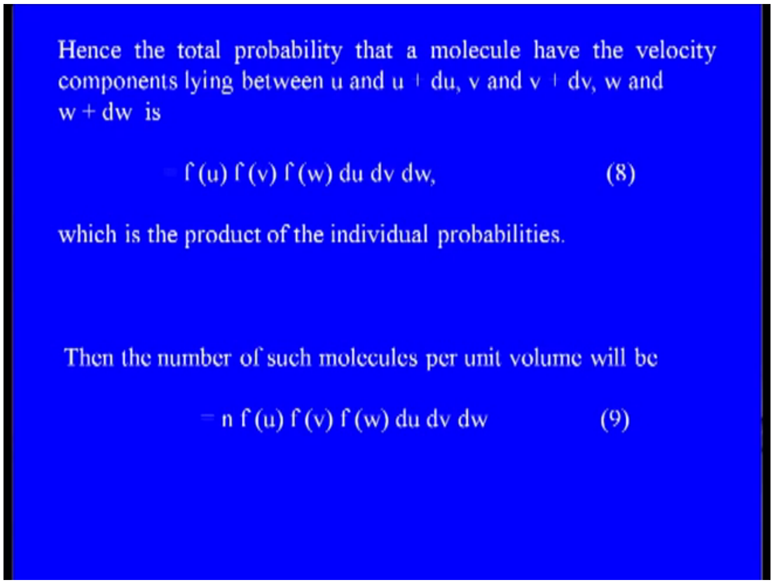

Hence the total probability that a molecule have the velocity components lying between u and u + du, v and v + dv, w and w + dw is

$$= f(u)\, f(v)\, f(w)\, du\, dv\, dw, \qquad (8)$$

which is the product of the individual probabilities.

Then the number of such molecules per unit volume will be

$$= n\, f(u)\, f(v)\, f(w)\, du\, dv\, dw \qquad (9)$$

So, let us say hence the total probability that a molecule can have the velocity lying between u and u + du, v and v + dv, w and w + dw = f of u times f of v times f of w du dv dw which is just the product of the probabilities three independent probabilities f of udu f of vdv f of wdw which is the product of the individual probability. Then the number of such molecules per unit volume will be n f of u f of v f of w du dv dw.

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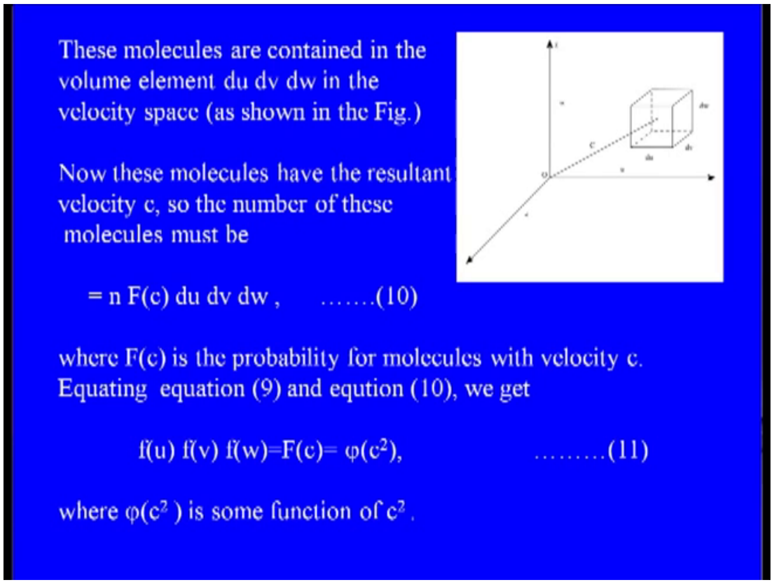

These molecules are contained in the volume element du dv dw in the velocity space (as shown in the Fig.)

Now these molecules have the resultant velocity c, so the number of these molecules must be

$$= n\,F(c)\,du\,dv\,dw\,, \qquad \ldots\ldots.(10)$$

where F(c) is the probability for molecules with velocity c. Equating equation (9) and eqution (10), we get

$$f(u)\,f(v)\,f(w)=F(c)=\varphi(c^2), \qquad \ldots\ldots\ldots(11)$$

where $\varphi(c^2)$ is some function of c^2.

These molecules are content in the volume element du dv dw in the velocity space. So, as shown in the figure from this figure you can see that these number of molecules are confined in the volume element du dv dw. Now these molecules have the resultant velocity c. So, the number of these molecules must be n into F of c du dv dw.

As I have already told you c is the resultant velocity which is nothing but the income in analogy to the position radius vector in the coordinate space c is nothing but the c square = u square + v square + w Square where F of c is the probability for molecules with velocity c. So, equating equation 9 and 10 we get f of u into f of v into f of w equal to some other function which is Fof c which is nothing but the Phi of c square say where Phi of c square is some function of c square.

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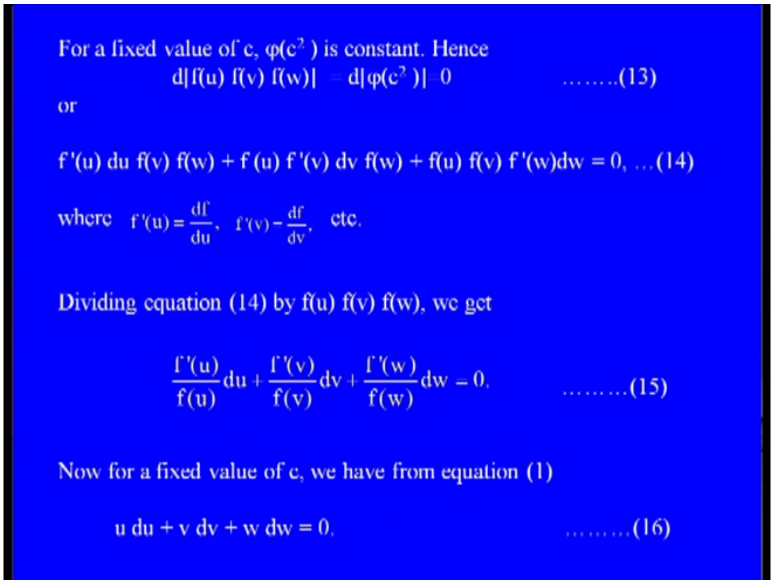

For a fixed value of c, $\varphi(c^2)$ is constant. Hence

$$d[f(u)\, f(v)\, f(w)] = d[\varphi(c^2)] = 0 \qquad \text{........(13)}$$

or

$$f'(u)\, du\, f(v)\, f(w) + f(u)\, f'(v)\, dv\, f(w) + f(u)\, f(v)\, f'(w) dw = 0, \quad \text{...(14)}$$

where $f'(u) = \frac{df}{du}, \quad f'(v) = \frac{df}{dv},$ etc.

Dividing equation (14) by f(u) f(v) f(w), we get

$$\frac{f'(u)}{f(u)} du + \frac{f'(v)}{f(v)} dv + \frac{f'(w)}{f(w)} dw = 0. \qquad \text{.........(15)}$$

Now for a fixed value of c, we have from equation (1)

$$u\, du + v\, dv + w\, dw = 0. \qquad \text{.........(16)}$$

So, for fixed values of c Phi of c square is constant, so d of Phi should be 0. So, if I will take if we will differentiate d of Phi, so what I will get d of f of u times f of d times f of w = 0, so if I will take if I will differentiate them keeping other two values constant. Suppose if I want to take different set u, so, keeping constant v and w. Similarly I can differentiate v keeping constant u and w, similarly I can differentiate w you keeping u and v constant.

So, if I will do these simple mathematics what we will get it f of f prime u du f of v f of w + f of u prime v dv f w + f of u f of v times f prime w dw = 0 where prime stands for the differentiation f prime u stands for df by du f prime v stands for df by dv keeping other two variable constant. Similarly f of f prime w means the derivative of probability function with respect to w keeping u and v constant.

So, dividing the equation 14 by f of u f of v f of w, we will get a beautiful equation f prime u by fu du + f prime v by fv dv + f prime w by fw dw = 0. Now for fixed values of c we know that u square c square = u square + v square + w square. If I differentiate this thing for a fixed values of c square d of c square should be 0 that means udu + vdv + wdw should be 0. Now I have two equations one equation is obtained by differentiating d of Phi = 0.

And second equation by demanding for a fixed values of c square d of c square = 0, so I got the two differential equation from there I have to find out what is f of u? What is f of v? And what is f of w? So, I have two differential equations two differential equation from there how to solve f of u f of v f of w. This is a new method which is known as Laplace's method of undetermined multiplier.

So, what to do suppose if you have one mother equation which is a prime udu just let me show you let me show you there is a equation 15 this is the mother equation other constant equation is equation number 16. So, what is the rule to do it, if you have 2, 3 constant equations so you multiply one constant to each constant condition and then you add with the mother equation. And then you demand that the coefficient of each variable will be 0 separately.

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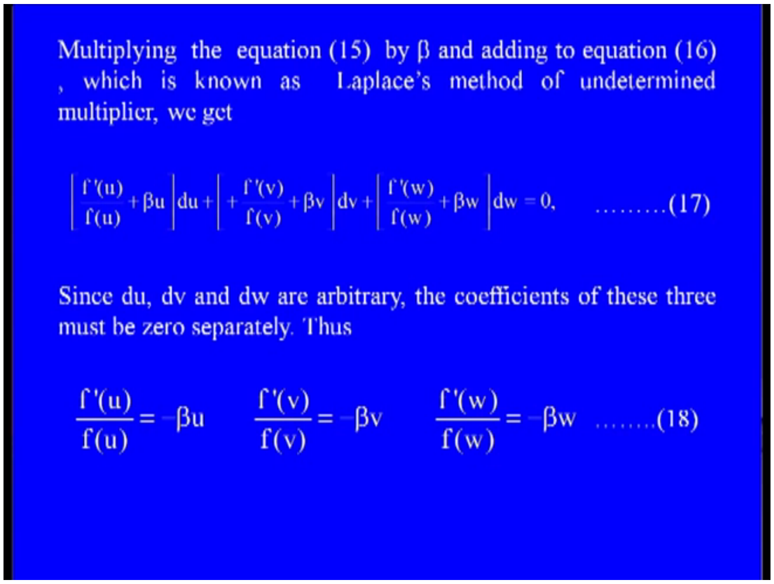

Multiplying the equation (15) by β and adding to equation (16), which is known as Laplace's method of undetermined multiplier, we get

$$\left[\frac{f'(u)}{f(u)}+\beta u\right]du+\left[\frac{f'(v)}{f(v)}+\beta v\right]dv+\left[\frac{f'(w)}{f(w)}+\beta w\right]dw=0, \qquad \ldots\ldots\ldots(17)$$

Since du, dv and dw are arbitrary, the coefficients of these three must be zero separately. Thus

$$\frac{f'(u)}{f(u)}=-\beta u \qquad \frac{f'(v)}{f(v)}=-\beta v \qquad \frac{f'(w)}{f(w)}=-\beta w \qquad \ldots\ldots(18)$$

Let me start it how to do it, so, if f prime u by fu + beta u du + f prime v by fb + beta vdv + f prime w by fw + beta wdu = 0. So, since I got this equation then I will demand that since du, dv and dw the the coefficient of du dv dw since they are arbitrary. So, the coefficient should be 0 independently if I will do it what I will get it? I will get first equation f prime u by fu + beta u du = 0 that means f prime u by fu + beta u = 0.

Similarly f prime v by fv + beta v = 0 and f prime w by fw + beta w = 0, so I got three ordinary partial differential equation if I will solve these three ordinary difference partial different ordinary differential equations or not this partial differential equation this is three ordinary but differential equation. Once I will solve these three ordinary differential equations, I will get the solution f of u, f of v, f of w respectively.

Once I got it then I will multiply fu fv fw to get f of c square ok. This is the algorithm of our mathematics. So, let us do it, so f prime u by fu = - beta u f prime v by fv = - beta v f prime w by fw = -beta w.

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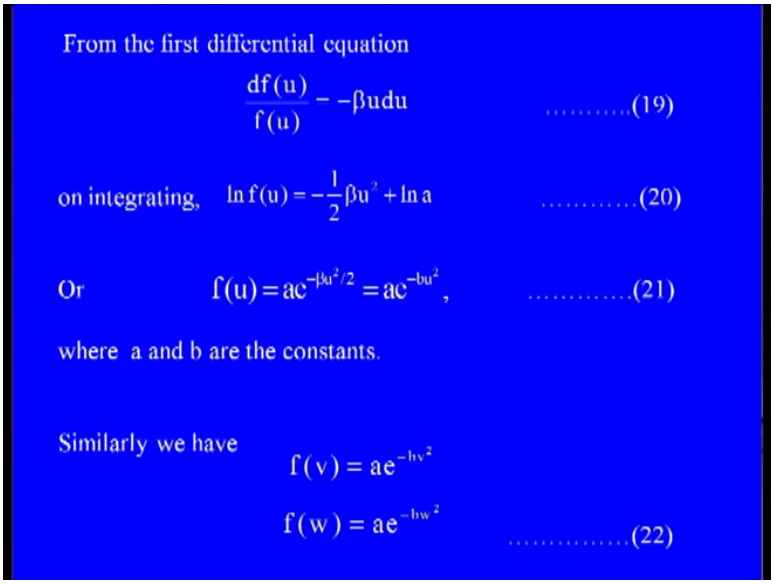

From the first differential equation

$$\frac{df(u)}{f(u)} = -\beta u du \qquad \text{...........(19)}$$

on integrating, $\ln f(u) = -\frac{1}{2}\beta u^2 + \ln a$ ....(20)

Or $f(u) = ae^{-\beta u^2/2} = ae^{-bu^2}$, ...(21)

where a and b are the constants.

Similarly we have

$$f(v) = ae^{-bv^2}$$

$$f(w) = ae^{-bw^2} \qquad \text{...............(22)}$$

So, from the first differential equation if I will solve it I will get log of f of u = - half beta u square + log a where a is the integration constant. If I will rewrite it then I will get it f of u = a to the power -beta u square by 2. Let us say beta by 2 is some other constant b. So, I will get a to the power - beta u square. So, where a and b are the constant. Similarly I can solve other two ordinary differential equations.

So, I will get f of v and f of w which will look like f of v = ae to the power -beta bv square f of w = ae to the power - beta bw square where b is nothing but the beta by 2 where a is nothing but the integration constant. So, once I got f of u, f of v, f of w then I can define the combined probability having the velocity u and u + du, v and v + dv and w and w+dw.

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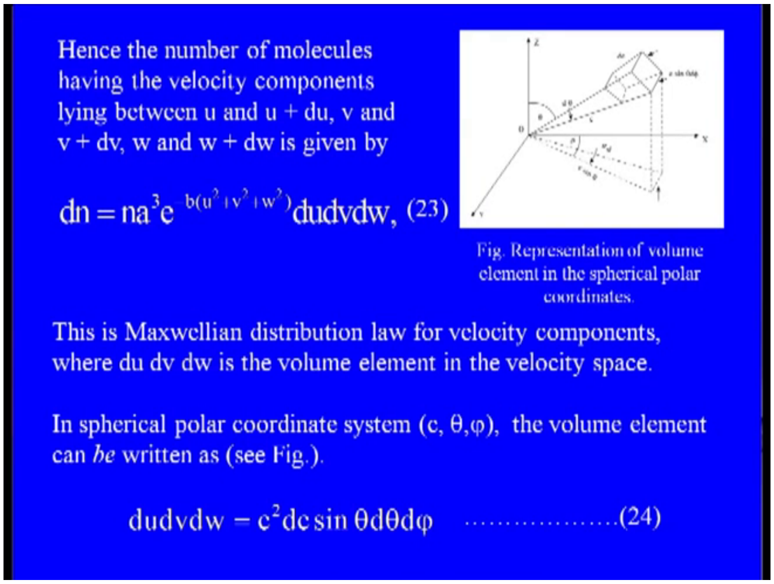

Hence the number of molecules having the velocity components lying between u and u + du, v and v + dv, w and w + dw is given by

$$dn = na^3 e^{-b(u^2+v^2+w^2)} dudvdw, \quad (23)$$

Fig. Representation of volume element in the spherical polar coordinates.

This is Maxwellian distribution law for velocity components, where du dv dw is the volume element in the velocity space.

In spherical polar coordinate system (c, θ,φ), the volume element can *be* written as (see Fig.).

$$dudvdw = c^2 dc \sin\theta d\theta d\varphi \quad \ldots\ldots\ldots\ldots\ldots\ldots(24)$$

So, hence the number of molecules having the velocity component lying between u and u + du, v and v + dv, w and w + dw is given as the product of this theta which I will get dn =n aq e to the power -bu square + v square + w square du dv dw. So, so this is the famous Maxwell distribution law of velocities where du, dv, dw is the volume element in the velocity space. In the spherical polar coordinate system in the velocity space c theta Phi the volume element can be written as du times dv times dw = c squared dc sine theta d theta d Phi.

As we know in the coordinate space the volume element is defined as dv = R R square, R square dR sine theta d theta d Phi only difference is that R will be replaced by the resultant velocity vector which is c okay.

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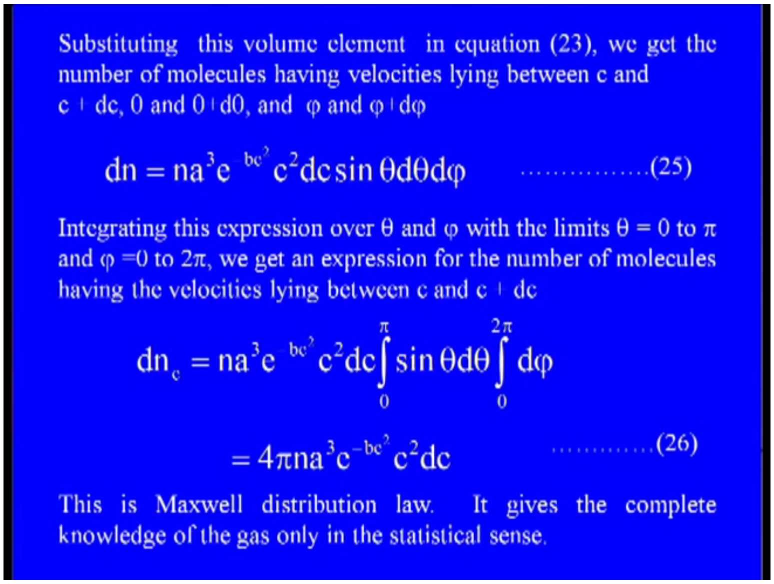

Substituting this volume element in equation (23), we get the number of molecules having velocities lying between c and c + dc, θ and θ + dθ, and φ and φ + dφ

$$dn = na^3e^{bc^2}c^2dc\sin\theta d\theta d\varphi \qquad \ldots\ldots\ldots\ldots\ldots(25)$$

Integrating this expression over θ and φ with the limits θ = 0 to π and φ =0 to 2π, we get an expression for the number of molecules having the velocities lying between c and c + dc

$$dn_c = na^3e^{bc^2}c^2dc\int_0^{\pi}\sin\theta d\theta\int_0^{2\pi}d\varphi$$

$$= 4\pi na^3c^{-bc^2}c^2dc \qquad \ldots\ldots\ldots\ldots(26)$$

This is Maxwell distribution law. It gives the complete knowledge of the gas only in the statistical sense.

So, if I will substitute the volume element du, dv, dw in the equation 23 we will get this dn = n aq e to the power -bc square c square dc sine theta d theta d Phi, so this is the this is nothing but the number of molecules having velocities lying between c and c + dc theta and theta + d theta and Phi and Phi + d Phi. If you will integrate this equation for theta, theta 0 to PI, Phi 0 to 2 pi what we will get it.

We will get dn c = n aq e to the -beta square c square dc, 0 to PI sine theta d theta, 0 to 2 pi d Phi what we will get 4 PI n aq e to the power - bc square c squared dc this is Maxwell distribution of law. It gives the complete knowledge of the gas only in this statistical sense. So, now let me discuss this thing. So, if we will rewrite this equation in terms of some new variable then things will be clear much and things will be much easier much clearer.

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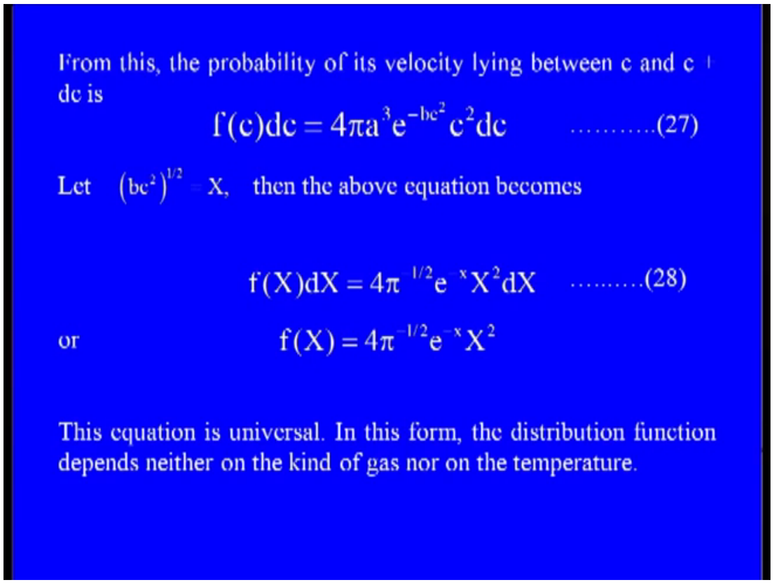

From this, the probability of its velocity lying between c and c + dc is

$$f(c)dc = 4\pi a^3 e^{-bc^2} c^2 dc \quad ....(27)$$

Let $(bc^2)^{1/2} = X$, then the above equation becomes

$$f(X)dX = 4\pi^{-1/2} e^{-X} X^2 dX \quad ....(28)$$

or $$f(X) = 4\pi^{-1/2} e^{-x} X^2$$

This equation is universal. In this form, the distribution function depends neither on the kind of gas nor on the temperature.

So, let me do it from this the probability of it is velocity lying between cu and c + dc it just you divide it by n what you will get f of c dc = 4 PI aq e to the power - bc square c square dc. Let us substitute X which is nothing but the square root of bc square. Then the above equation reduces to f of X dX = 4 pi to the power minus of e to the power -X X square dX or f of X equal to 4 PI to the power minus of e to the power - X square.

So, this is the probability having some quantity of X where X is nothing but the velocity but it is a new variable. So, f of X is nothing but 4PI to the power of minus of e to the -X X square. This equation is universal in the sense in this form the distribution function depends neither on the kind of gas or nor on the temperature.

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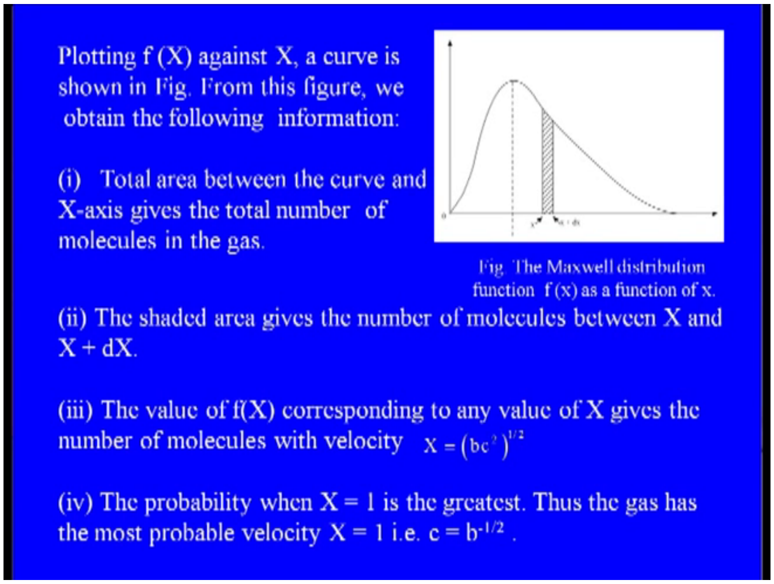

Plotting f (X) against X, a curve is shown in Fig. From this figure, we obtain the following information:

(i) Total area between the curve and X-axis gives the total number of molecules in the gas.

Fig. The Maxwell distribution function f (x) as a function of x.

(ii) The shaded area gives the number of molecules between X and X + dX.

(iii) The value of f(X) corresponding to any value of X gives the number of molecules with velocity $X = (bc^2)^{1/2}$

(iv) The probability when X = 1 is the greatest. Thus the gas has the most probable velocity X = 1 i.e. $c = b^{-1/2}$.

So, now if you will plot f of X versus X, a curve is shown in the figure from this figure we will obtain the following information. So, if you plot f of X versus X the curve whatever we obtain; I will get the following information by plotting this curve. So, I will tell one by one. The total area between the curve and the X-axis gives the total number of molecules in the gas. So, this is the definition of integration.

Integration f of X dX a to b will give you the area under the curve which is nothing but the result okay. The shaded area which is shown in the figure gives the number of molecules having the variables between X and X +dX or you can say in terms of velocity shaded area gives the number of molecules having velocities some c to c + dc. The values of fX corresponding to any values of X gives the number of molecules with velocities X = square root of bc square.

The probability when X =1 is the greatest thus the gas has the most probable velocity X = 1 from there you will get c = 1 by square root of d so these are the features of this curve. So, taking this who will compute the constants a and b by normalizing it. Once I will get a and b then I will substitute it back values of a and b. We will get complete Maxwell distribution law of velocities.

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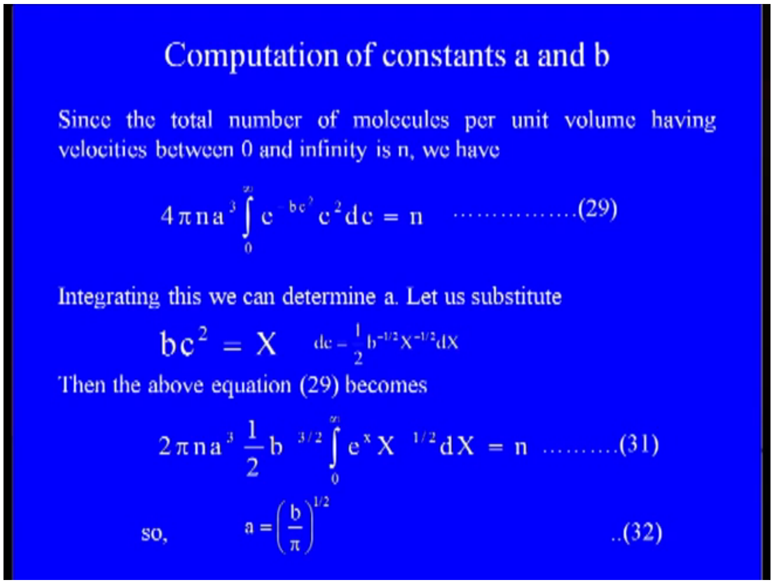

Computation of constants a and b

Since the total number of molecules per unit volume having velocities between 0 and infinity is n, we have

$$4\pi n a^3 \int_0^\infty e^{-bc^2} c^2 dc = n \quad \text{.................}(29)$$

Integrating this we can determine a. Let us substitute

$$bc^2 = X \qquad dc = \frac{1}{2} b^{-1/2} X^{-1/2} dX$$

Then the above equation (29) becomes

$$2\pi n a^3 \frac{1}{2} b^{3/2} \int_0^\infty e^{x} X^{1/2} dX = n \quad \text{..........}(31)$$

so, $$a = \left(\frac{b}{\pi}\right)^{1/2} \quad \text{..}(32)$$

Since the total number of molecules per unit volume having velocities 0 to infinity is n, so we will integrate it we will get the total number of molecules. So, if you will integrate it the 4 Pi naq will come out of this integration 0 to infinity e to the - bc square c square dc = n, so, if I integrate it this is nothing but a Gaussian type integration. If you will integrate it by substituting bc square = X.

So, then I will rewrite this equation in terms of new variables what I will get it 2 Pi naq half b to the power -3 by 2, 0 to infinity e to the power X, X to the power of minus of dX = n. From there if I will put the value of 0 to infinity e to the power X e to the power X X to the or minus of dX then I will get a = b by PI to the power 1.

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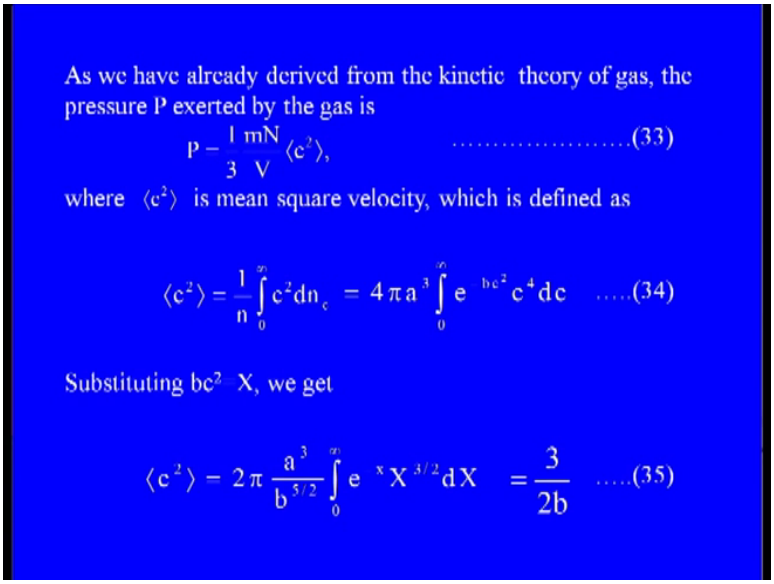

As we have already derived from the kinetic theory of gas, the pressure P exerted by the gas is

$$P = \frac{1}{3}\frac{mN}{V}\langle c^2\rangle, \qquad \ldots\ldots\ldots\ldots\ldots\ldots(33)$$

where $\langle c^2\rangle$ is mean square velocity, which is defined as

$$\langle c^2\rangle = \frac{1}{n}\int_0^\infty c^2 dn_c = 4\pi a^3 \int_0^\infty e^{-bc^2} c^4 dc \qquad \ldots..(34)$$

Substituting $bc^2 = X$, we get

$$\langle c^2\rangle = 2\pi \frac{a^3}{b^{5/2}} \int_0^\infty e^{-X} X^{3/2} dX = \frac{3}{2b} \qquad \ldots..(35)$$

So, as we have already derived from the kinetic theory of gases the pressure P exerted by the gas is P = one third mN by V average values of c Square where c square is the mean square velocity which is defined as c square average is 1 by n 0 to infinity c square dn c, so if you will substitute the values of dn c if you substitute the values of dn c from this equation from the equation 26 and substitute it there then what we will get it.

Average values of c square we will get 4 Pi aq 0 to infinity e to the power -bc square c to the power 4 dc because c square is already here another c square you will come from dn c. So, c square times c square is c to the power 4. Then again I will substitute bc square X, so I will rewrite the equation for the mean square velocity is 2 Pi aq b to the by b to the power 5 by 2, 0 to infinity e to the power –X, X to the power 3 by 2 DX which comes out to be 3 by 2d.

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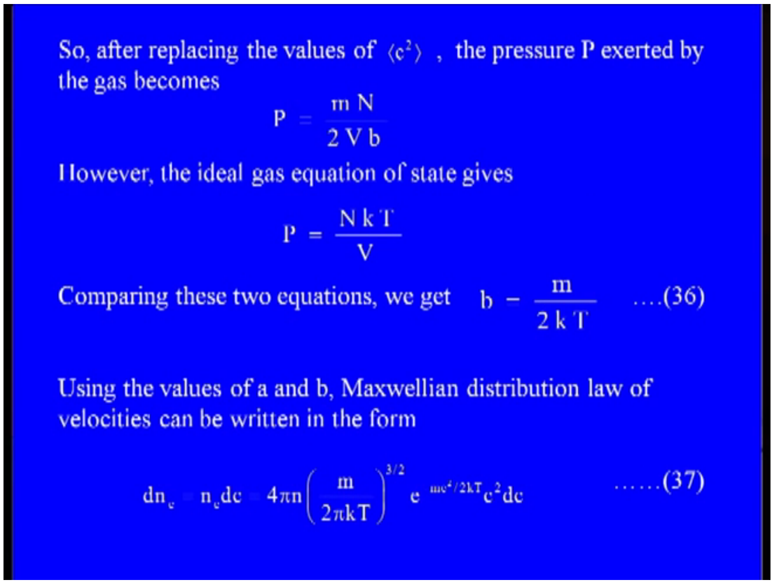

So, after replacing the values of $\langle c^2 \rangle$, the pressure P exerted by the gas becomes

$$P = \frac{m N}{2 V b}$$

However, the ideal gas equation of state gives

$$P = \frac{N k T}{V}$$

Comparing these two equations, we get $b = \frac{m}{2 k T}$ ….(36)

Using the values of a and b, Maxwellian distribution law of velocities can be written in the form

$$dn_c = n_c dc = 4\pi n \left(\frac{m}{2\pi k T} \right)^{3/2} e^{-mc^2/2kT} c^2 dc \qquad \text{……(37)}$$

Since if you will substitute average values of square velocity c square in the pressure equation, so we will get P = small m times N by 2 Vb. However the ideal equation of state gives P = Nkt by V, if I compare these two equations one is from the experimental result P NkT by V and P = mN by 2 Vb, if you this is this equation is obtained from the Maxwell kinetic theory of gases.

If you compare this two equation you will get the values of b. Once you will get the values of b then we have already derived it some equation in terms of a and b you substitute b, so you will get the values of b. So, in principle we have obtained the constants a and b. So, b = M by 2kT. So, if you will substitute a and b then Maxwell distribution law of velocities can be written in the form dn c del 2 nc dc = 4 PIn m by 2Pi kT e to the power 3 by 2 e to the power - mc square by k2 kT c square dc.

This is the famous Maxwell Boltzmann distribution law this tells the number of molecules having velocities c and c + dc, this is a very famous equation and probability you just divide it by Nc you will get the probability of finding a molecule between c and c + dc which is nothing but 4 Pi N will go away m by 2P kT eto the power 3 by 2 e to the power -mc square by 2 kT into c square dc.

Here I should tell some interesting thing although it is a heuristic argument e to the power some any exponential function with a negative e to the -mc square by 2 kT. You can think in different ways. There are two energy scale in the system one is since it the system is at thermal equilibrium, so obviously there is a temperature. So, there is one scale which is related to the thermal energy scale which is of the order of kT.

Other energy scale which is intrinsic to the energy or to the particle which is nothing but the simple kinetic energy half mV square, whenever there is a dominant, whenever there is a computation of two scales here it is the energy scale so there will be always some exponential function will come which is the ratio of the two energy scale mc square by 2kT.

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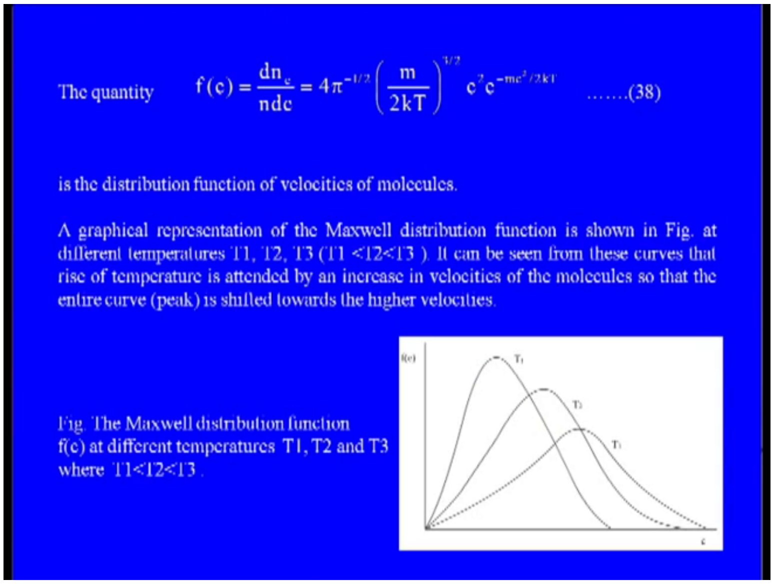

The quantity $f(c) = \frac{dn_c}{ndc} = 4\pi^{-1/2}\left(\frac{m}{2kT}\right)^{3/2} c^2 e^{-mc^2/2kT}$ …….(38)

is the distribution function of velocities of molecules.

A graphical representation of the Maxwell distribution function is shown in Fig. at different temperatures T1, T2, T3 (T1 <T2<T3). It can be seen from these curves that rise of temperature is attended by an increase in velocities of the molecules so that the entire curve (peak) is shifted towards the higher velocities.

Fig. The Maxwell distribution function f(c) at different temperatures T1, T2 and T3 where T1<T2<T3 .

The quantity f of c which is I have already told you dnc by ndc which is 4 Pi to the power minus of m by 2KT to the 3 by 2c square e to the -mc square by 2 KT is the distribution function of velocities of molecules. So, if you plot these Maxwell distributions function for 3 different temperature of the system. Suppose some temperature having thermal equilibrium at temperature T1 other system having temperature T2 obviously is in thermal equilibrium.

Other system is in also obviously in thermal equilibrium having temperature T2, if you plot these three figures for the three different systems. So, what you will see if you will plot f of c versus c having temperature T1, T2, T3 who are T1 less than T2 less than T3 what you will see the peak of this curve we will slip to the higher temperature. So, physically what does it mean if you, if this system has a system or in a system is in a higher temperature.

So, obviously probability of finding molecules having large kinetic energy is large which is obvious because the square of the mean of the square of the energy below energy mean energy is proportional to the temperature of the system. If the temperature is large so mean energy will be large so that is the reason the peak of the curve will be shifted towards in the right direction.

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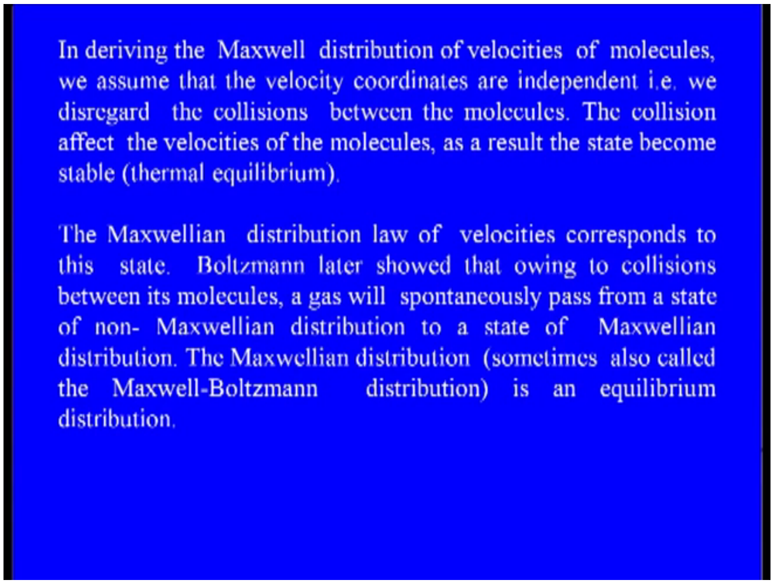

In deriving the Maxwell distribution of velocities of molecules, we assume that the velocity coordinates are independent i.e. we disregard the collisions between the molecules. The collision affect the velocities of the molecules, as a result the state become stable (thermal equilibrium).

The Maxwellian distribution law of velocities corresponds to this state. Boltzmann later showed that owing to collisions between its molecules, a gas will spontaneously pass from a state of non- Maxwellian distribution to a state of Maxwellian distribution. The Maxwellian distribution (sometimes also called the Maxwell-Boltzmann distribution) is an equilibrium distribution.

So, now in deriving the Maxwell distribution of velocities of molecules, we assume that the velocity coordinates are independent that is we disregard the collision between the molecules. The collision affects the velocities of the molecules as a result the state becomes stable or thermal equilibrium. So, that means somebody could ask that why, when thermal equilibrium will be achieved.

So, in I have seen I have given in two lectures in the kinetic theory of gases where I have told somewhere that temperature is a concept temperature is a concept of the isotropicity in the momentum space. Suppose you have taken n number of molecules in a container of gas. Initially they are momentum in different directions; maybe it is different but after some time there will be some isotropic in the momentum space which is average values of PX square should be equal to the average values of PY square equal to average values of PZ square.

This isotropicity momentum space is known as the thermal equilibrium. But in my earlier two lectures with this equilibrium situation has been obtained by the collision of a molecule between between the molecules and the wall of the container, not the collision among themselves but what Maxwell's told the if the collisions will be more if the collisions will be more frequent the system will achieve thermal equilibrium more quickly okay.

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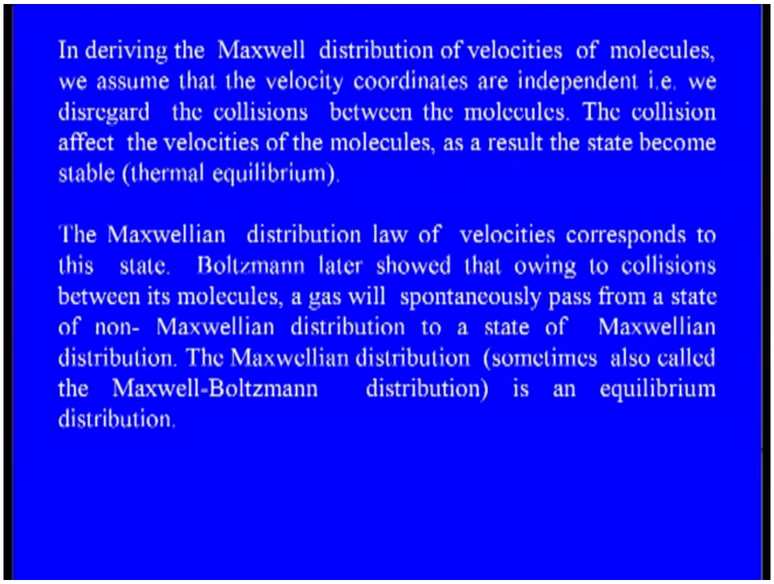

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The Maxwell distribution law of velocities corresponds to this state Boltzmann were later sort that going to collisions between its molecules a gas will spontaneously pass from a state of non maxwellian distribution to a state of maxwellian distribution. The maxwellian distribution sometimes also called the Maxwell Boltzmann distribution is an equilibrium distribution. So, now I will tell;

Since it is a very beautiful laws of nature so one must verify the Maxwell distribution law in a couple of minutes in a couple of minutes I will try to demonstrate some experiment through who is it has been proved now that the disc Maxwell distribution is perfect law who is which is obeyed by the molecules in a gas okay. Initially whenever people have performed some experiment they are experiment in some sense crude so their experimental error is not perfect.

They are the matches which they experience the agreement between theory and experiment up to 15 percent of error. So, more and more refined experiments have been done through this refined experiment now it is confirmed that there is no deviation between theory and experiment.

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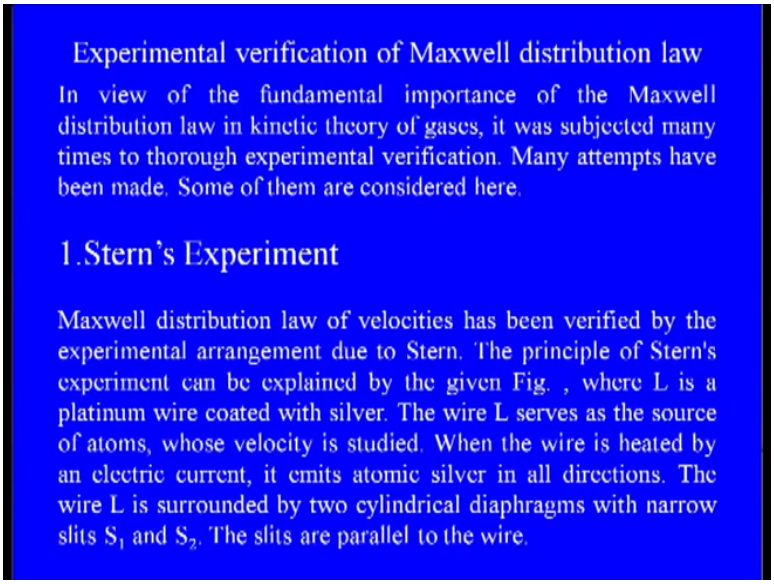

Experimental verification of Maxwell distribution law

In view of the fundamental importance of the Maxwell distribution law in kinetic theory of gases, it was subjected many times to thorough experimental verification. Many attempts have been made. Some of them are considered here.

1.Stern's Experiment

Maxwell distribution law of velocities has been verified by the experimental arrangement due to Stern. The principle of Stern's experiment can be explained by the given Fig. , where L is a platinum wire coated with silver. The wire L serves as the source of atoms, whose velocity is studied. When the wire is heated by an electric current, it emits atomic silver in all directions. The wire L is surrounded by two cylindrical diaphragms with narrow slits S_1 and S_2. The slits are parallel to the wire.

So, in view of the fundamental importance of the Maxwell distribution law in kinetic theory of gases it was subjected many times to thorough experimental verification. Many attempts have been made some of them are considered here. So, first experiment in this endeavor Stern experiment. So, let me start it. Maxwell distribution of velocities has been verified by the experimental arrangement due to Stern.

The principle of Stern experiment can be expressed by the given figure where L is a platinum or let me show you the experiment this is the schematic diagram of the experiment where L is a platinum wear coated with silver. The wire L serves as the source of atoms whose velocity is to be studied when the wire is heated by an electric current it emits atomic silver in all direction. The wire L is surrounded by two cylindrical diaphragms with narrow slits S1 and S2.

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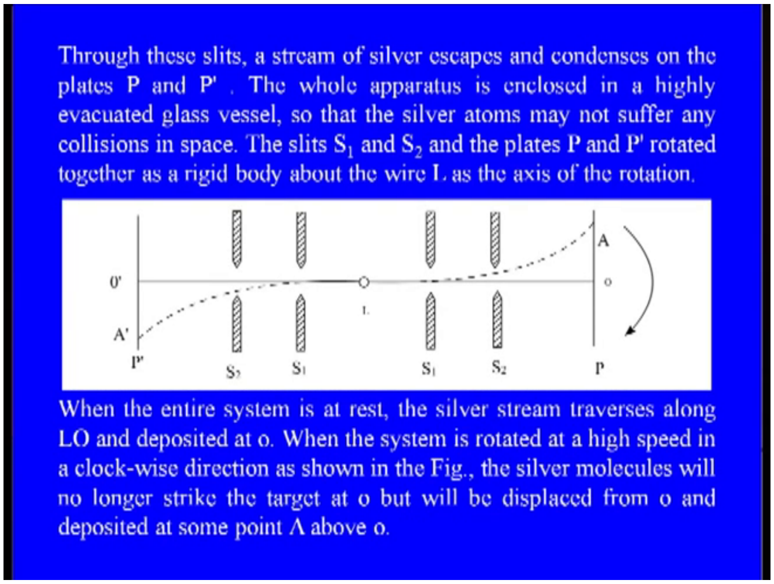

Through these slits, a stream of silver escapes and condenses on the plates P and P' . The whole apparatus is enclosed in a highly evacuated glass vessel, so that the silver atoms may not suffer any collisions in space. The slits S_1 and S_2 and the plates P and P' rotated together as a rigid body about the wire L as the axis of the rotation.

When the entire system is at rest, the silver stream traverses along LO and deposited at o. When the system is rotated at a high speed in a clock-wise direction as shown in the Fig., the silver molecules will no longer strike the target at o but will be displaced from o and deposited at some point A above o.

Through these slits are parallel to the wire. Now this is the schematic diagram of this experimental arrangement through these slits is stream of silver escapes and condenses on the plates P and P prime okay. The whole apparatus is enclosed in a highly evacuated glass vessel so that the silver atoms may not suffer in the collisions in the space. That means in the within the experimental vessel it is maintained at a very high vacuum.

So, that there are no air molecules inside. So, that while silver atoms foil it is going to the 2P and P prime, so it will not suffer any collision. So, that is the reason it has kept in an evacuated chamber okay. These slits S1 and S2 and the plates P and P Prime rotated together as a rigid body about the wire L as the axis of rotation. When the entire system is at rest the silver stream traverses along LO obviously and ad and deposited at O.

Because it will straight hit to O, so obviously it will be deposited at the point O. When the system is rotated around the axis of the wire so at high speed in clockwise direction which is shown in the figure the silver molecules will no longer strike the target at O but will be displaced from the from O and deposited at some point above O.

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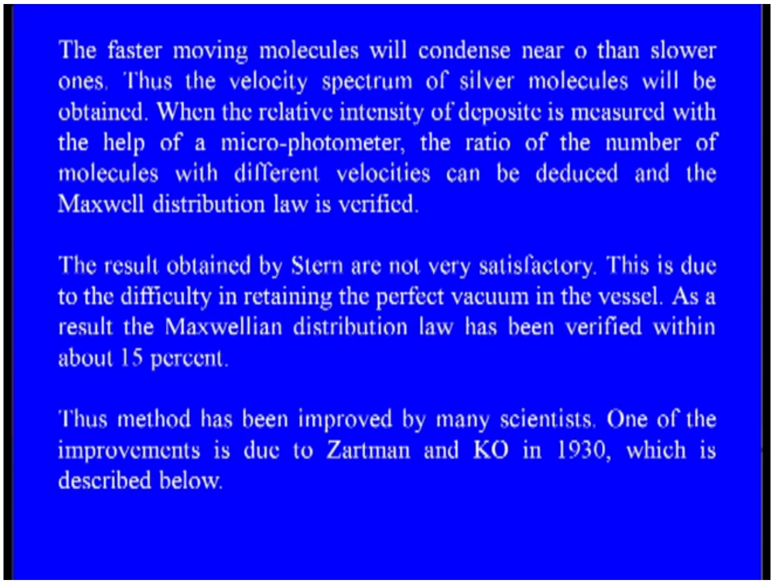

The faster moving molecules will condense near o than slower ones. Thus the velocity spectrum of silver molecules will be obtained. When the relative intensity of deposite is measured with the help of a micro-photometer, the ratio of the number of molecules with different velocities can be deduced and the Maxwell distribution law is verified.

The result obtained by Stern are not very satisfactory. This is due to the difficulty in retaining the perfect vacuum in the vessel. As a result the Maxwellian distribution law has been verified within about 15 percent.

Thus method has been improved by many scientists. One of the improvements is due to Zartman and KO in 1930, which is described below.

The faster moving molecules will condense near to narrow then slower ones. Thus the velocity spectrum of silver molecules will be obtained when the relative intensity of deposit is measured with the help of a micro photometer. The ratio of the number of molecules with different velocities can be deduced and the Maxwell distribution law is verified. However the result obtained by the Stern experiment is quite satisfactory.

But still there are some problems in the experiment because although I have kept it in a highly evacuated chamber but still that time the vacuum technology is not so in advanced situation, so you cannot make it completely evacuated. So, that is the reason the agreement between the theory and experiment a tail up to 15 percent.

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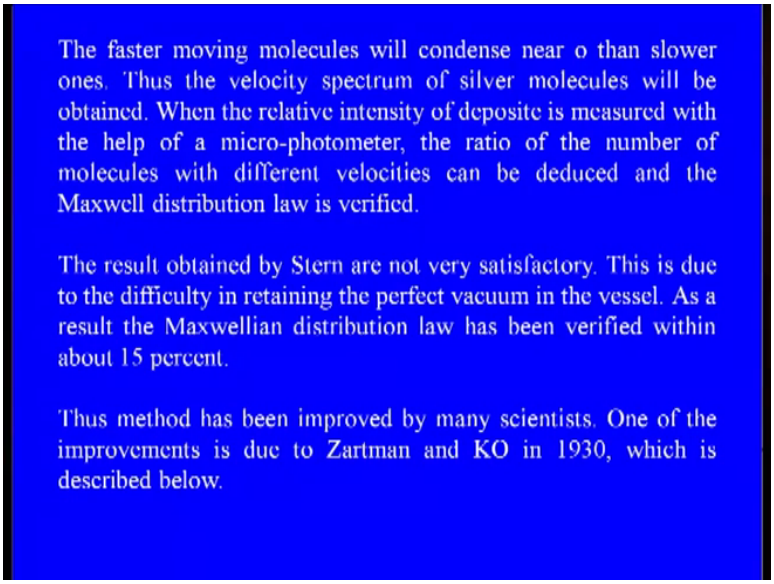

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So and this is due to the difficulty in retaining the perfect vacuum in the vessel as just I have told you. As a result the Maxwell distribution law has been verified within about 15%. So, thus this method needs to be improved so exactly this method has been improved by many scientists. One of the improvements is due to Zartman and KO in 1930 which is described below.

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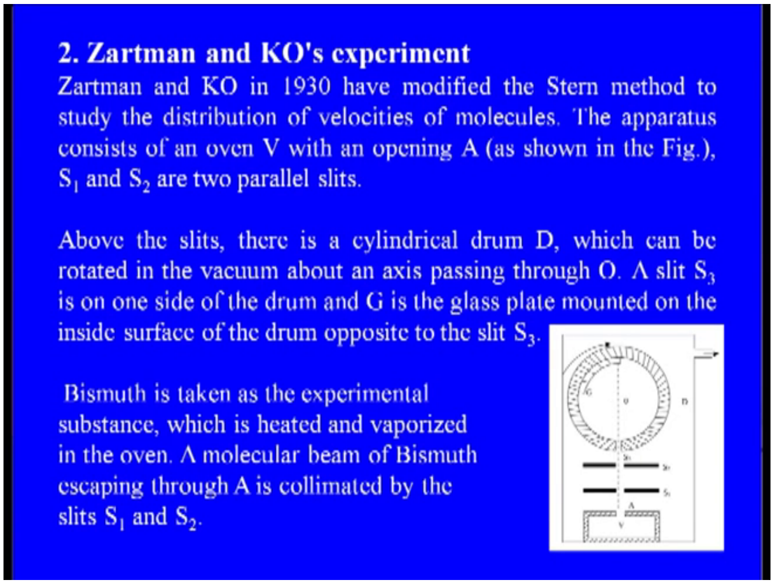

2. Zartman and KO's experiment

Zartman and KO in 1930 have modified the Stern method to study the distribution of velocities of molecules. The apparatus consists of an oven V with an opening A (as shown in the Fig.), S_1 and S_2 are two parallel slits.

Above the slits, there is a cylindrical drum D, which can be rotated in the vacuum about an axis passing through O. A slit S_3 is on one side of the drum and G is the glass plate mounted on the inside surface of the drum opposite to the slit S_3.

Bismuth is taken as the experimental substance, which is heated and vaporized in the oven. A molecular beam of Bismuth escaping through A is collimated by the slits S_1 and S_2.

First I will explain Zartman and KO’s experiment, Zartman and KO in 1930 have modified the Stern method to study the distribution of velocities of molecules. The apparatus consists of an oval V with a narrow opening a which is shown in this figure S1 and S2 are the two parallel slit's through which these through which these ah molecules will go.

Above the slits there is a cylindrical drum D which can be obviously here some section of the cylindrical drum is shown and which is nothing but a circle. Which can be rotated in the vacuum about an axis passing through O, it is given in the figure. A slit S3 is on is on one side of the drum and G is the glass plate mounted on the inside surface of the drum opposite to the S3 okay. So, you can see and that S3 is just opposite to the glass plate G.

It is shown in the figure, so bismuth is taken as the experimental substance in the earlier experiment we have taken in the Stern experiment we have taken silver. But here they have taken these working substance as the bismuth. So, bismuth is taken as the experimental substance which is heated and vaporized in the oven. A molecular beam of bismuth escaping through a through this narrow opening is collimated by these slits S1 and S2.

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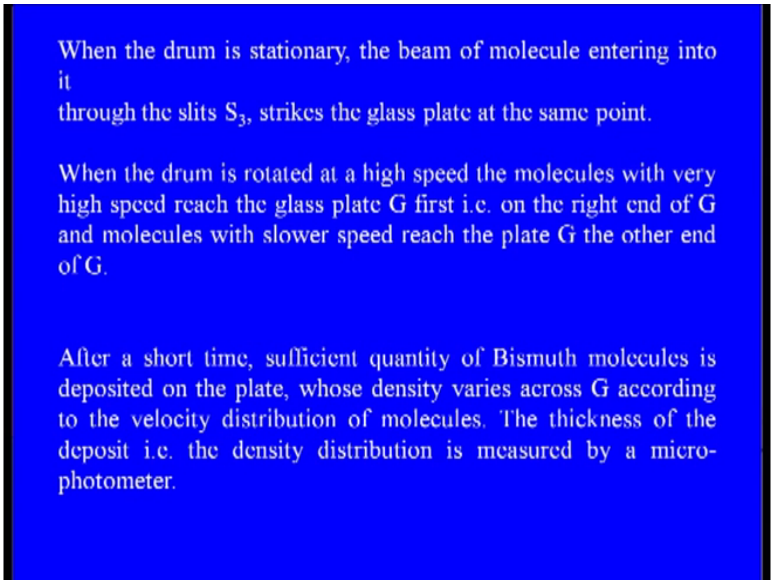

When the drum is stationary, the beam of molecule entering into it
through the slits S_3, strikes the glass plate at the same point.

When the drum is rotated at a high speed the molecules with very high speed reach the glass plate G first i.e. on the right end of G and molecules with slower speed reach the plate G the other end of G.

After a short time, sufficient quantity of Bismuth molecules is deposited on the plate, whose density varies across G according to the velocity distribution of molecules. The thickness of the deposit i.e. the density distribution is measured by a micro-photometer.

So, finally when the drum is stationary the beam of the molecule entering into it through the slits S3 strike the glass plate at the same point. Obviously since it is at stationery it will go straight and hit the point in the drum. But when the drum is rotated at a high speed the molecules with very high speed reach the glass speed G first that is on the right hand of G and molecules with this slower speed reach the plate G on the other end of G.

So, after a short time sufficient quantity of bismuth molecules is deposited on the plate whose density varies across G, according to the velocity distribution of molecules the thickness of the deposit that is the density distribution is measured by a micro photometer.

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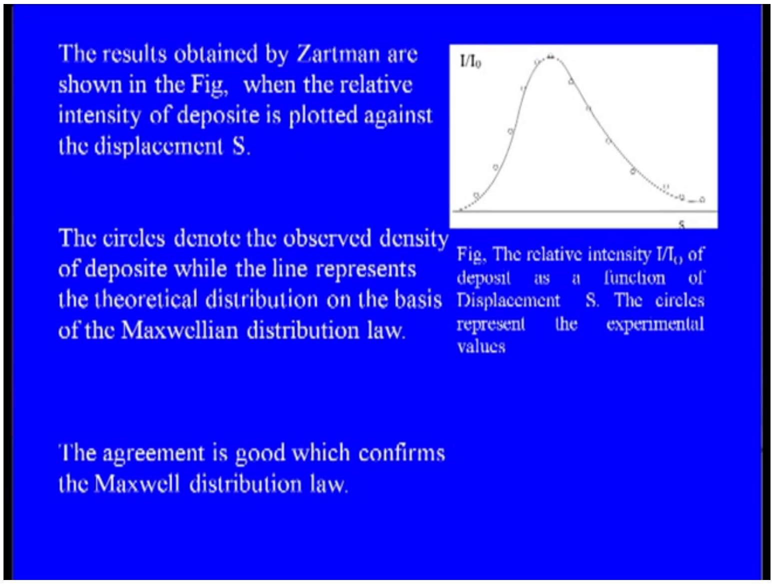

The results obtained by Zartman are shown in the Fig, when the relative intensity of deposite is plotted against the displacement S.

The circles denote the observed density of deposite while the line represents the theoretical distribution on the basis of the Maxwellian distribution law.

Fig, The relative intensity I/I_0 of deposit as a function of Displacement S. The circles represent the experimental values

The agreement is good which confirms the Maxwell distribution law.

And this result is obtained by Zartman and KO exactly matches with this almost matches not exactly almost matches with the theoretical distribution which is given by the solid line if you plot the intensity versus the this displacement is the dot of this straight line the straight these connected line gives the theoretical distribution and this circle this open circle is the result obtained by this experiment.

It almost matches with except at some high velocity and at the low velocity regime. So, these circles denote the observed density of deposit while the line represents the theoretical distribution on the basis of Maxwell distribution law so the experiment is good which confirms the Maxwell distribution law.